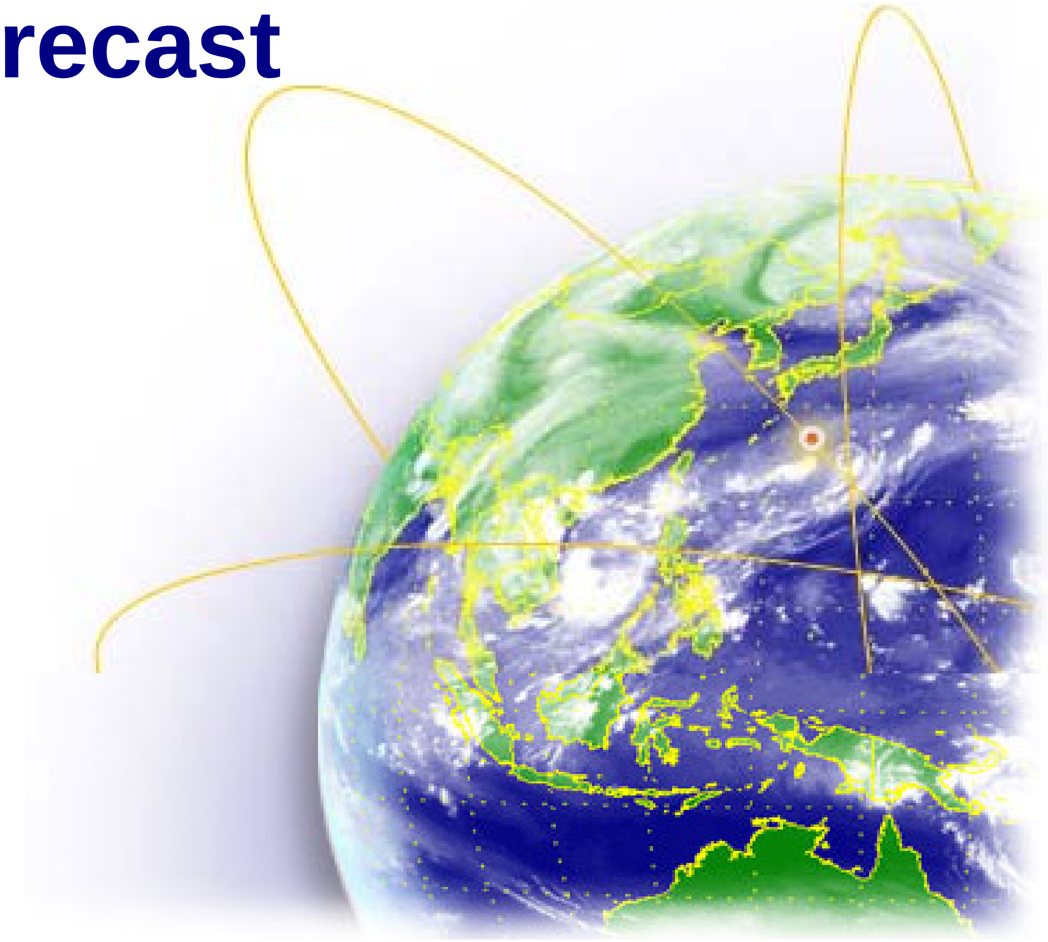
Introduction to Seasonal Multi-Model Ensemble Forecast



Saji N. Hameed

Content

1. APCC Multi-Model Ensemble

2. Multi-Model Ensemble Vs. Single Model

3. Multiple Regression Based Methods

4. Verification

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APCC Participating Models

Member Economies	Acronym	Organization	Model Resolution
Australia	POAMA	Bureau of Meteorology Research Centre	T47L17
Canada	MSC	Meteorological Service of Canada	1.875° × 1.875° L50
China	NCC	National Climate Center/CMA	T63L16
	IAP	Institute of Atmospheric Physics	4° × 5° L2
Chinese Taipei	CWB	Central Weather Bureau	T42L18
Japan	JMA	Japan Meteorological Agency	T63L40
Korea	GDAPS/KMA	Korea Meteorological Administration	T106L21
	GCPS/SNU	Seoul National University	T63L21
	METRI/KMA	Meteorological Research Institute	4° × 5° L17
Russia	MGO	Main Geophysical Observatory	T42L14
	HMC	Hydrometeorological Centre of Russia	1.12° × 1.4° L28
USA	IRI	International Research Institute	T42L19
	COLA	Center for Ocean-Land-Atmosphere Studies	T63L18
	NCEP	NCEP Coupled Forecast System	T62L64
	NSIPP/NASA	National Aeronautics and Space Administration	2° × 2.5° L34

Member Models -Hindcast

No.	Model	Hindcast data
1	CWB	1979-2004
2	MGO	1979-2004
3	NCEP	1981-2003
4	JMA	1983-2003
5	IRI	1979-2005
6	IRIF	1979-2005
7	HMC	1979-2003
8	NCC	1979-2005
9	GDAPS_O	1979-2003
10	GDAPS_F	1979-2003
11	GCPS	1979-2003
12	METRI	1979-2004
13	MSC	1969-2001
14	COLA	1981-2002
15	NASA	1993-2004
16	POAMA	1987-2001
17	IAP	1979-1999

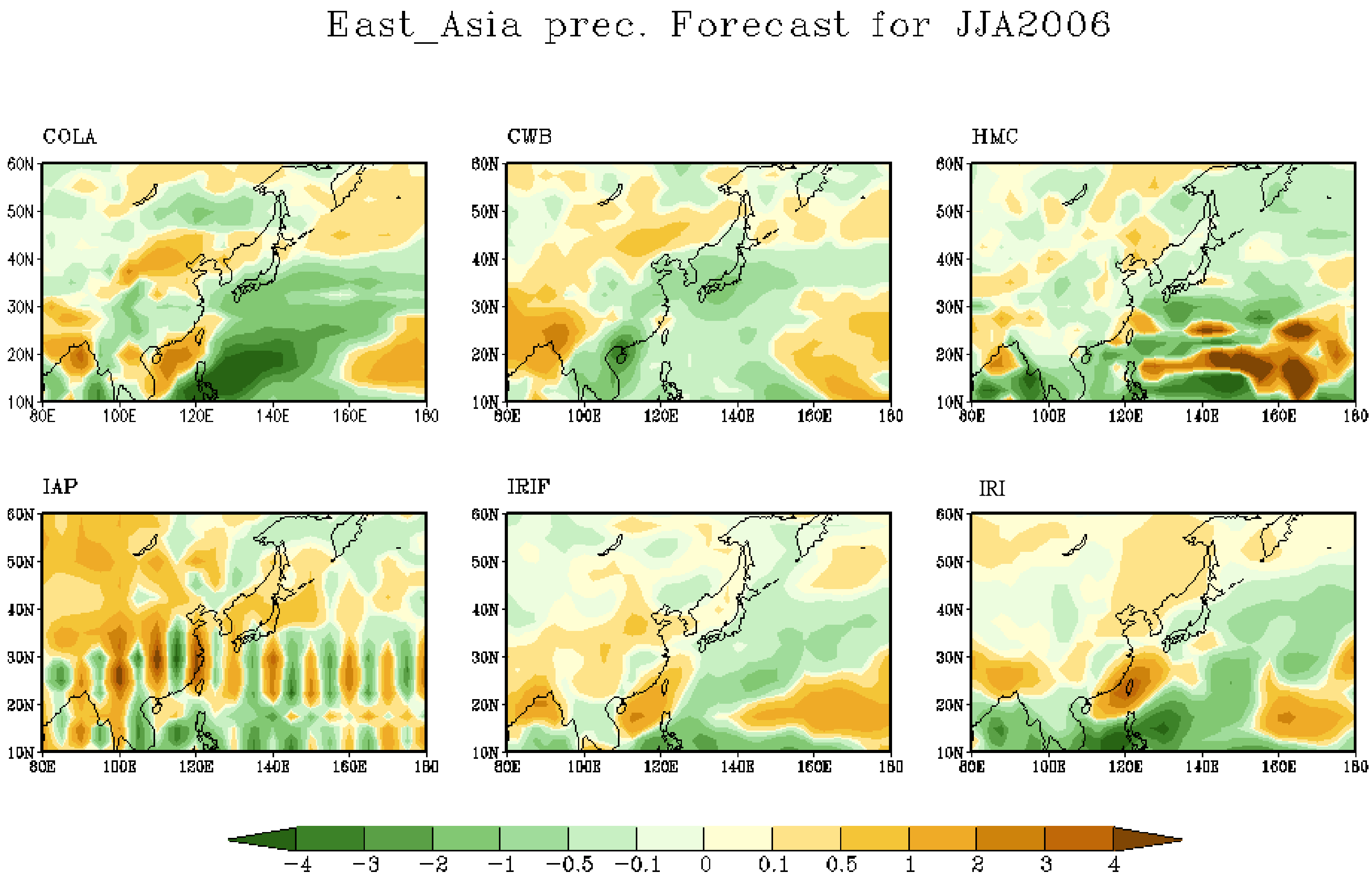
Training period:
1983 – 2003

(21 years)

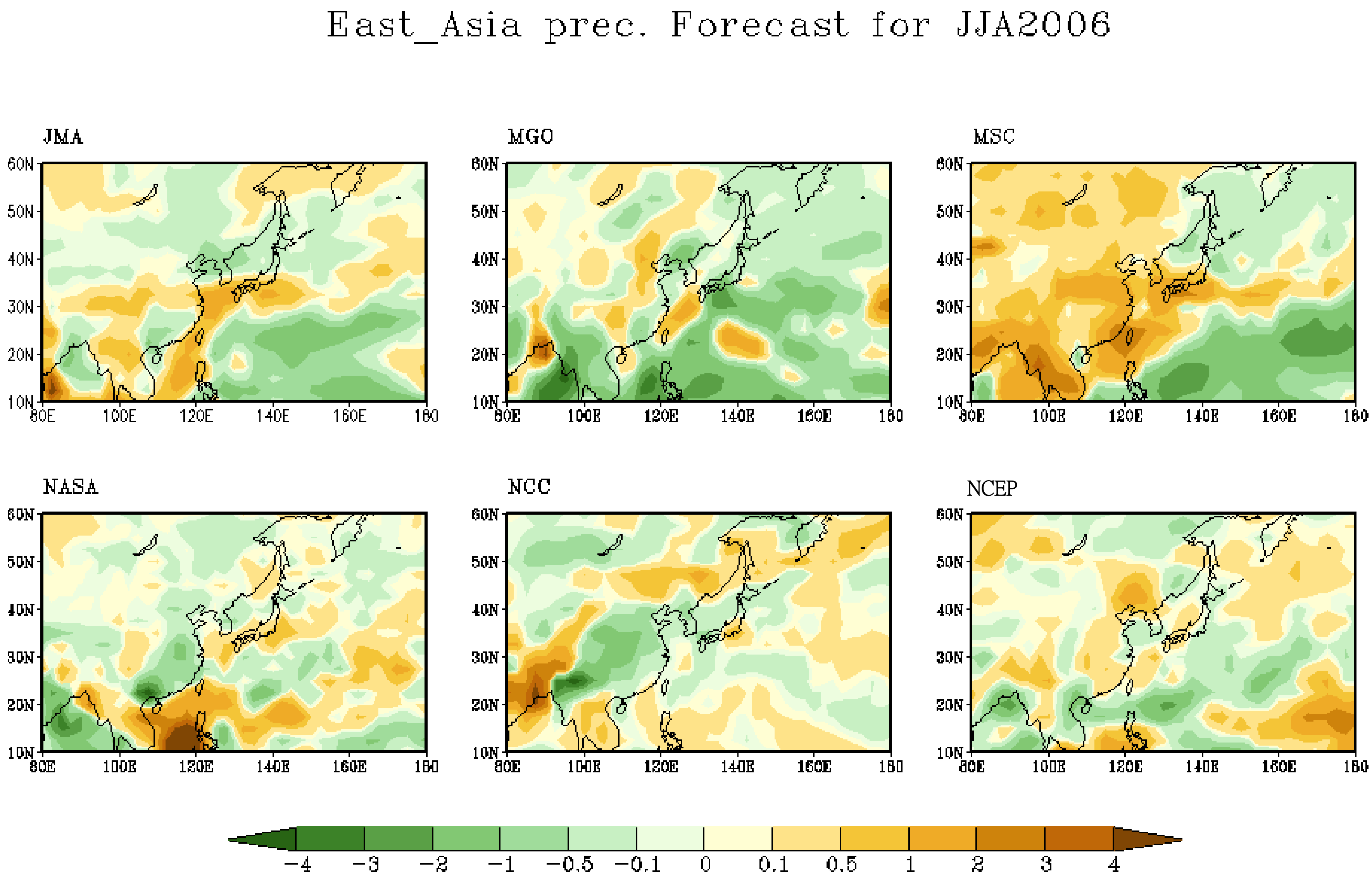
Short training

Quality Check

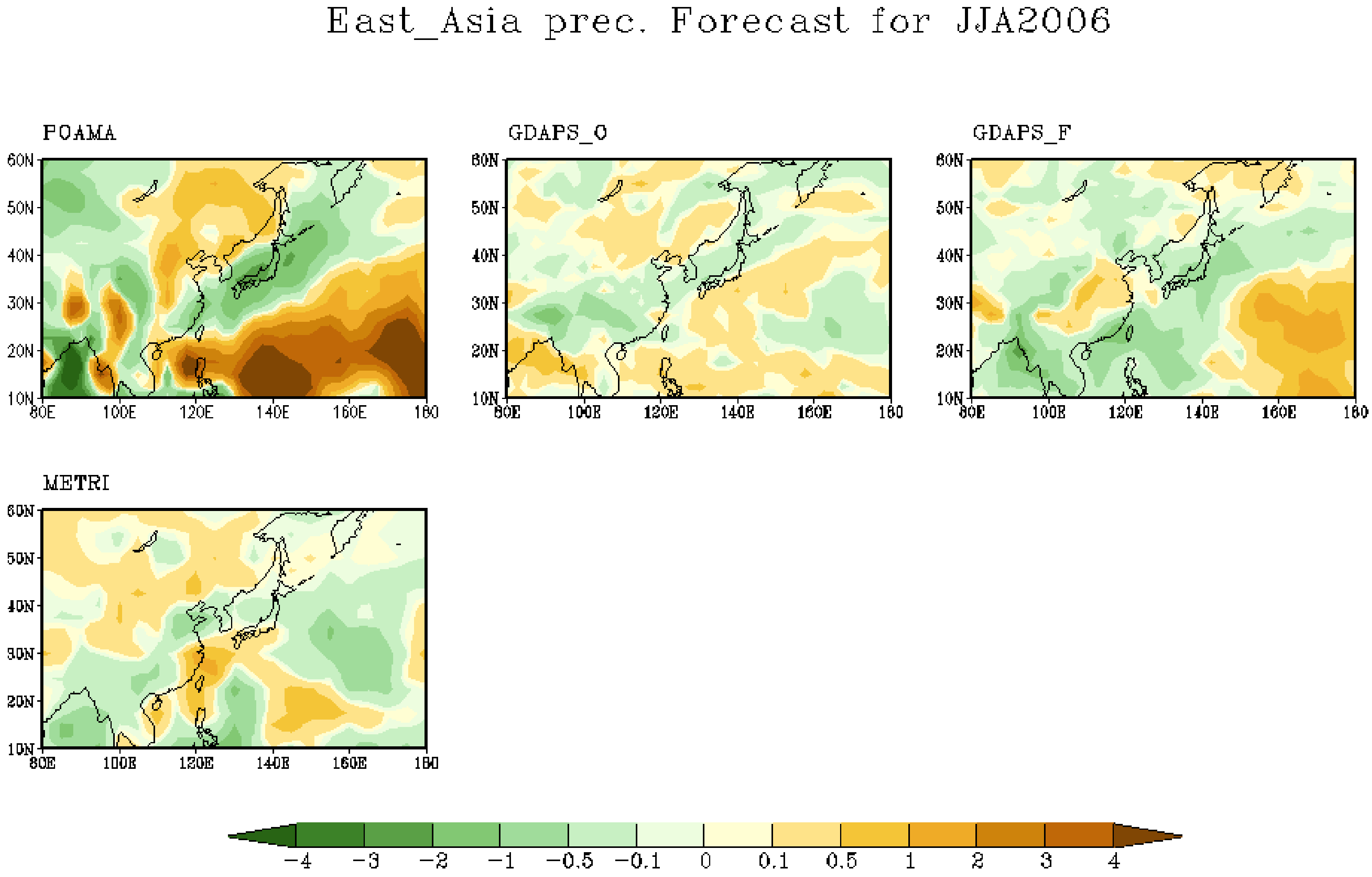
Precipitation Forecast in JJA 2006 (East Asia)



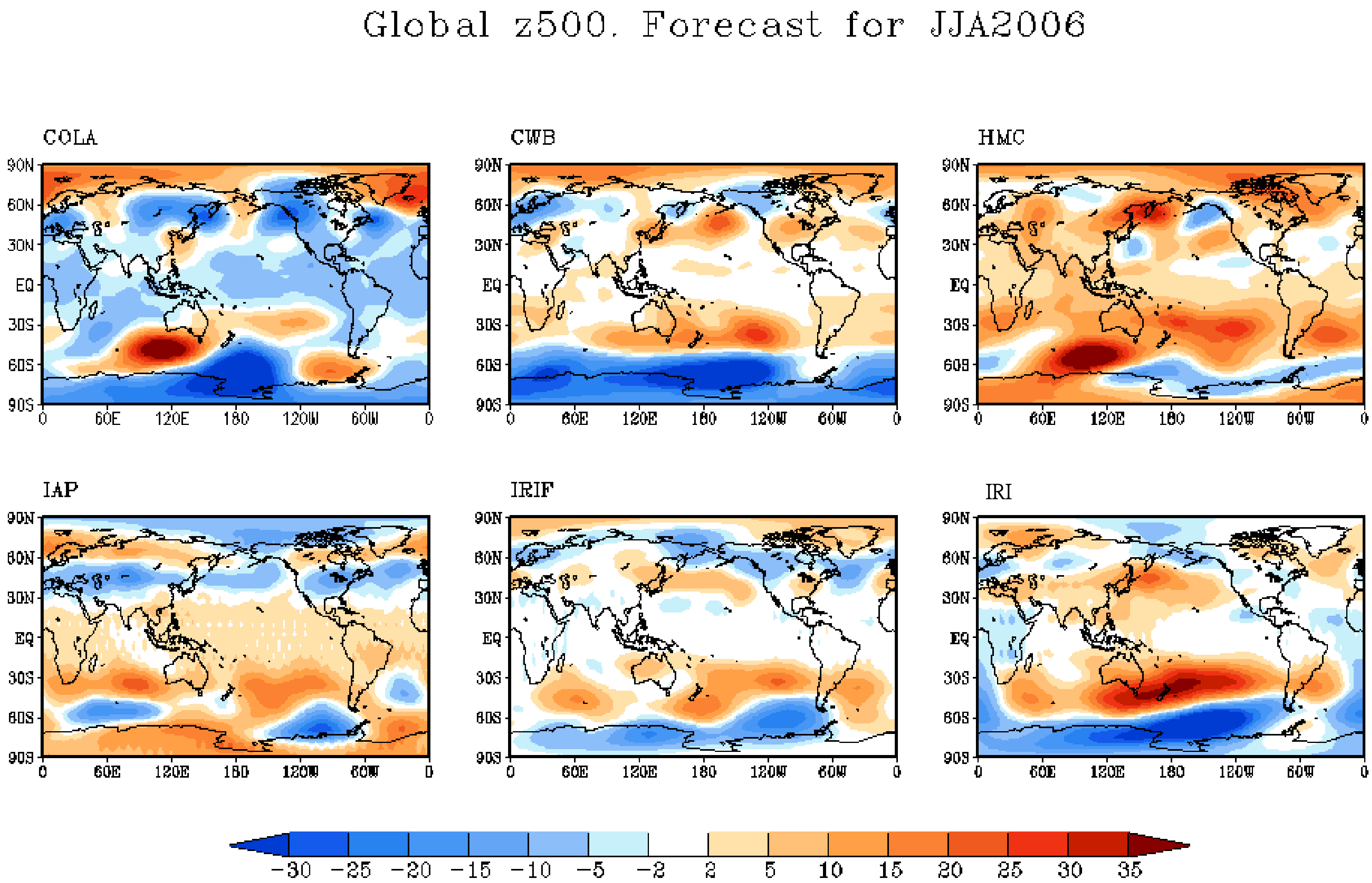
Precipitation Forecast in JJA 2006 (East Asia)



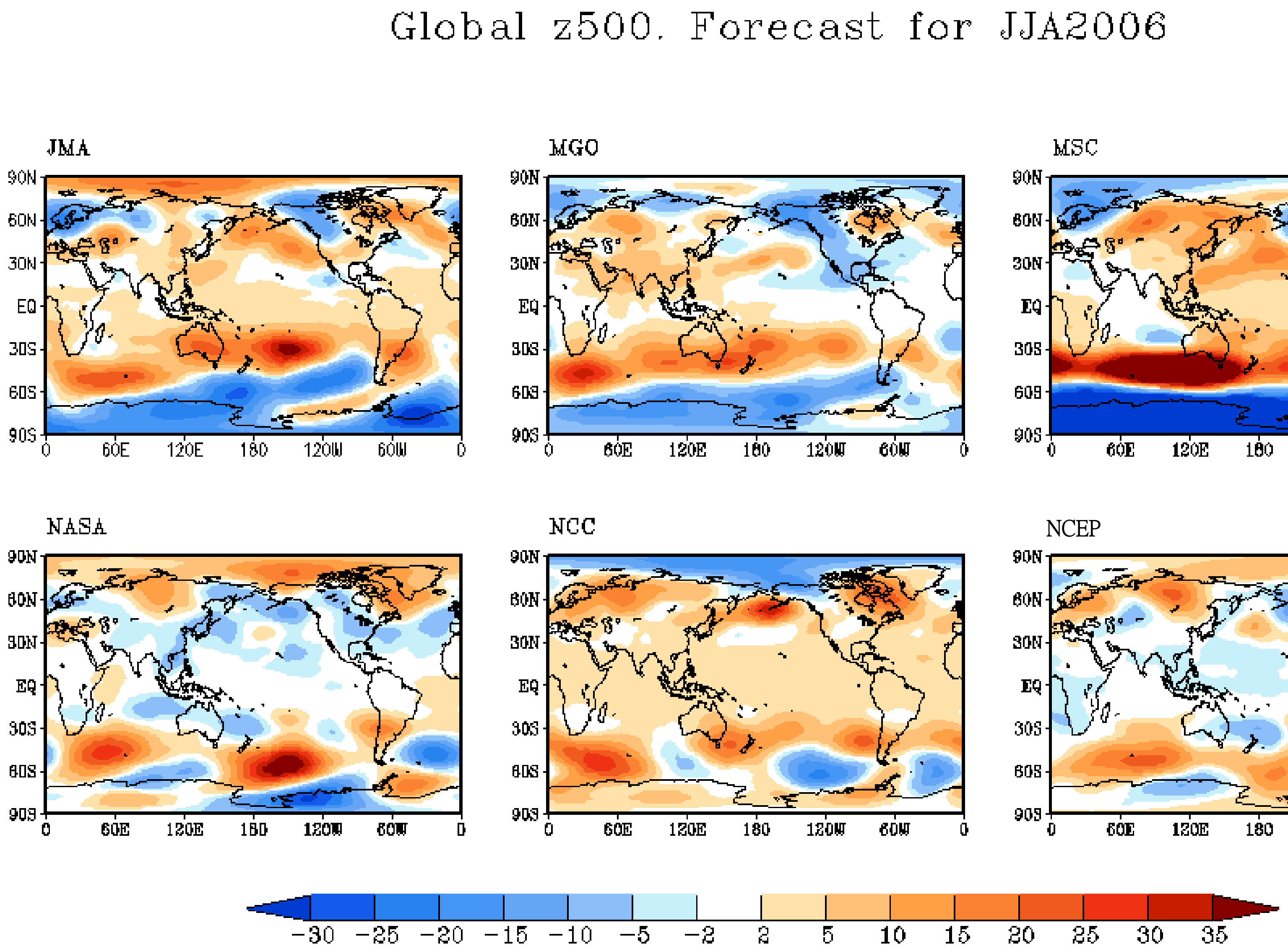
Precipitation Forecast in JJA 2006 (East Asia)



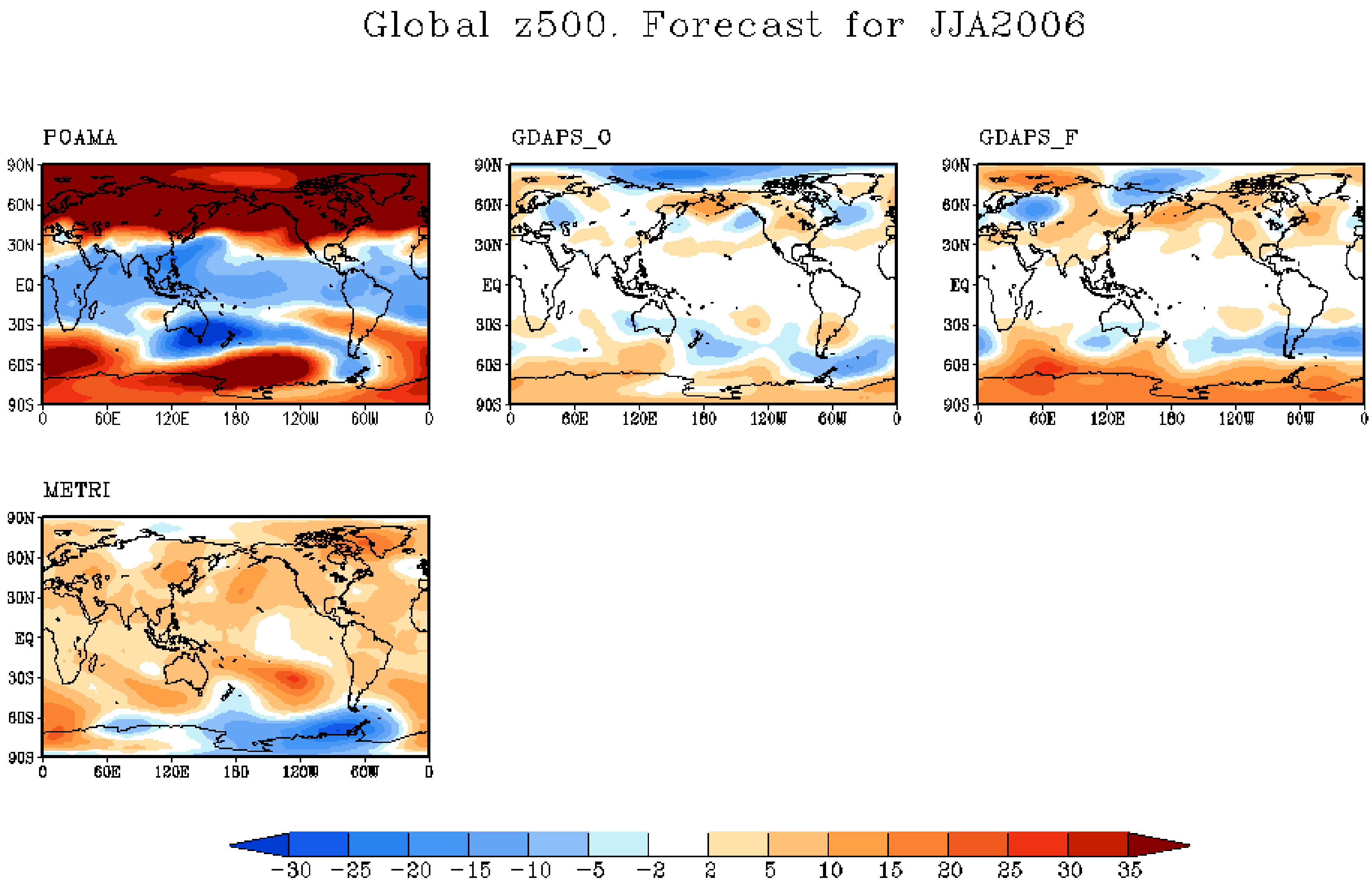
Z500 Forecast in JJA 2006 (Global)



Z500 Forecast in JJA 2006 (Global)



Z500 Forecast in JJA 2006 (Global)



2. Model Ensemble Vs. Single Model

Notations:

$\mathbf{Y}$ – $y_1, y_2, \ldots, y_n$ – series of observations

$\mathbf{X}_{\boldsymbol{i}}$ – $x_{i1}, x_{i2}, \ldots, x_{in}$ – series of *i*-model forecasts

We have *m* models and *n* years of forecasts

$\mathbf{Z}$ – $z_1, z_2, \ldots, z_n$ – series of MME-combined forecasts

$\mathbf{Z} = \mathbf{Z}(\mathbf{X})$

$$\mathbf{Y} = \mathbf{Z} + \mathbf{error}$$

MME – simple composite

$\mathbf{Z} = (\mathbf{X}_1+\mathbf{X}_2+..+\mathbf{X}_m)/m$

Let us compare MSE of **Y** Vs. $\mathbf{X}_i$ and **Y** Vs. **Z**

MSE of a single model forecast:

$$MSE_{YX_i} = \frac{1}{n}\sum_{j=1}^{n}(y_j - x_{ij})^2 = \sigma_Y^2 - 2r_{YX_i} + \sigma_{X_i}^2$$

Let us assume $\sigma_Y=\sigma_{Xi}=\sigma=1$

$$MSE_{YX_i} = 2\sigma^2(1-r_{YX_i}) = 2(1-r_{YX_i})$$

MME – simple composite

$\mathbf{Z} = (\mathbf{X}_1+\mathbf{X}_2+..+\mathbf{X}_m)/m$

Let us assume m=2 and $\sigma_Y=\sigma_{Xi}=\sigma=1$

MSE of a two model composite:

$$MSE_{YZ} = \frac{1}{n}\sum_{j=1}^{n}(y_j - (x_{1j} + x_{2j})/2)^2$$
$$= \sigma_Y^2 - \mathrm{cov}(Y, X_1) - \mathrm{cov}(Y, X_2) + \sigma_{X_1}^2/4 + \sigma_{X_2}^2/4 + \mathrm{cov}(X_1, X_2)/2$$
$$= 1.5 - r_{YX_1} - r_{YX_2} + r_{X_1X_2}/2$$

MME – simple composite

$\mathbf{Z} = (\mathbf{X}_1+\mathbf{X}_2+..+\mathbf{X}_m)/m$

MSE of a single model Vs. two model composite:

$$MSE_{YX_i} = 2(1 - r_{YX_i})$$

$$MSE_{YZ} = 1.5 - r_{YX_1} - r_{YX_2} + r_{X_1X_2}/2$$

MME – simple composite

$\mathbf{Z} = \mathbf{X}_1 + \mathbf{X}_2 + .. + \mathbf{X}_m$

$$cor(Y,Z) = \frac{\overline{\operatorname{cov}(Y, X_{i=1...m})}}{\sigma_Y \sigma_Z}$$

MME – simple composite

Let m = 2 **=>** **Z** = $\mathbf{X}_1 + \mathbf{X}_2$

$$cor(Y:Z) = \frac{r_1\sigma_1 + r_2\sigma_2}{\sqrt{\sigma_1^2 + 2r_{12}\sigma_1\sigma_2 + \sigma_2^2}}$$

$r_i = cor(\mathbf{Y},\mathbf{X}_i)$
$\sigma_i = \mathrm{STD}(\mathbf{X}_i)$

Let us assume: $\sigma_1 = \sigma_2$

$$cor(Y:Z) = \frac{r_1 + r_2}{\sqrt{2(1 + r_{12})}}$$

=> if $r_{12} < 1$, cor(**Y:Z**)>(r_1+r_2)/2

MME – simple composite

$$\text{cor}(\mathbf{Y:Z}) > (r_1 + r_2)/2$$

3. Multiple Regression Based Methods

Notations:

$\mathbf{Y}$ – $y_1, y_2, \ldots, y_n$ – series of observations

$\mathbf{X}_{\boldsymbol{i}}$ – $x_{i1}, x_{i2}, \ldots, x_{in}$ – series of *i*-model forecasts

We have *m* models and *n* years of forecasts

$\mathbf{Z}$ – $z_1, z_2, \ldots, z_n$ – series of MME-combined forecasts

$\mathbf{Y} = \mathbf{Z} + \mathbf{error}$

Multiple regression

Superensemble

Multiple regression

$$z_j = \sum_{i=1}^{m} b_i x_{ji}$$

i – model (1…m)
j – year (1…n)

Multiple regression

$$\mathbf{y = Xb + err}$$

y – vector (1…n) of observed anomalies
X – matrix (1…n, 1…m) of model forecasts
b – vector (1…m) of regression coefficients
err – vector (1…n) of errors

We are to minimize **err**

Multiple regression

We minimize **err** applying the Least Squares Estimation

Sum of squared errors:

$$Q = \mathbf{err}^{T}\mathbf{err} = (\mathbf{y}\text{-}\mathbf{Xb})^{T}(\mathbf{y}\text{-}\mathbf{Xb}) =$$

$$= \mathbf{y}^{T}\mathbf{y} - \mathbf{b}^{T}\mathbf{X}^{T}\mathbf{y}^{T} - \mathbf{y}^{T}\mathbf{Xb} + \mathbf{b}^{T}\mathbf{X}^{T}\mathbf{Xb} =$$

$$= \mathbf{y}^{T}\mathbf{y} - 2\mathbf{b}^{T}\mathbf{X}^{T}\mathbf{y} + \mathbf{b}^{T}\mathbf{X}^{T}\mathbf{Xb}$$

Multiple regression

Derivative of *Q* in respect to **b** is:

$$-2\mathbf{X}^T\mathbf{y} + 2\mathbf{X}^T\mathbf{X}\mathbf{b}$$

Condition of extremum is *Q* = 0, so that

$$\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

Singular value decomposition of $\mathbf{X}^{\mathrm{T}}\mathbf{X} = USV^{\mathrm{T}}$

S – diagonal matrix of eigenvalues

$U = V$ - eigenvectors of matrix $\mathbf{X}^{\mathrm{T}}\mathbf{X}$
(U=V because matrix $\mathbf{X}^{\mathrm{T}}\mathbf{X}$ is symmetric)

Regression coefficients:

$$\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\,\mathbf{X}^T\mathbf{y}$$

If to decompose $(\mathbf{X}^T\mathbf{X})^{-1}$ applying SVD,
above equation becomes very simple:

$$\mathbf{b} = \; VS^{-1}U^T\,\mathbf{X}^T\mathbf{y}$$

Accurate solution is when all m elements are non-zero but there may appear collinearity. Inaccurate but more stable solution is when some number of diagonal elements (M0<M) of S^{-1} are set to zero.

Synthetic Data

PCA+Multiple regression

$$O(x,t) = \sum_n \widetilde{O}_n(t)\phi_n(x)$$

$$F_i(x,T) = \sum_n \widetilde{F}_{i,n}(T) \cdot \varphi_{i,n}(x)$$

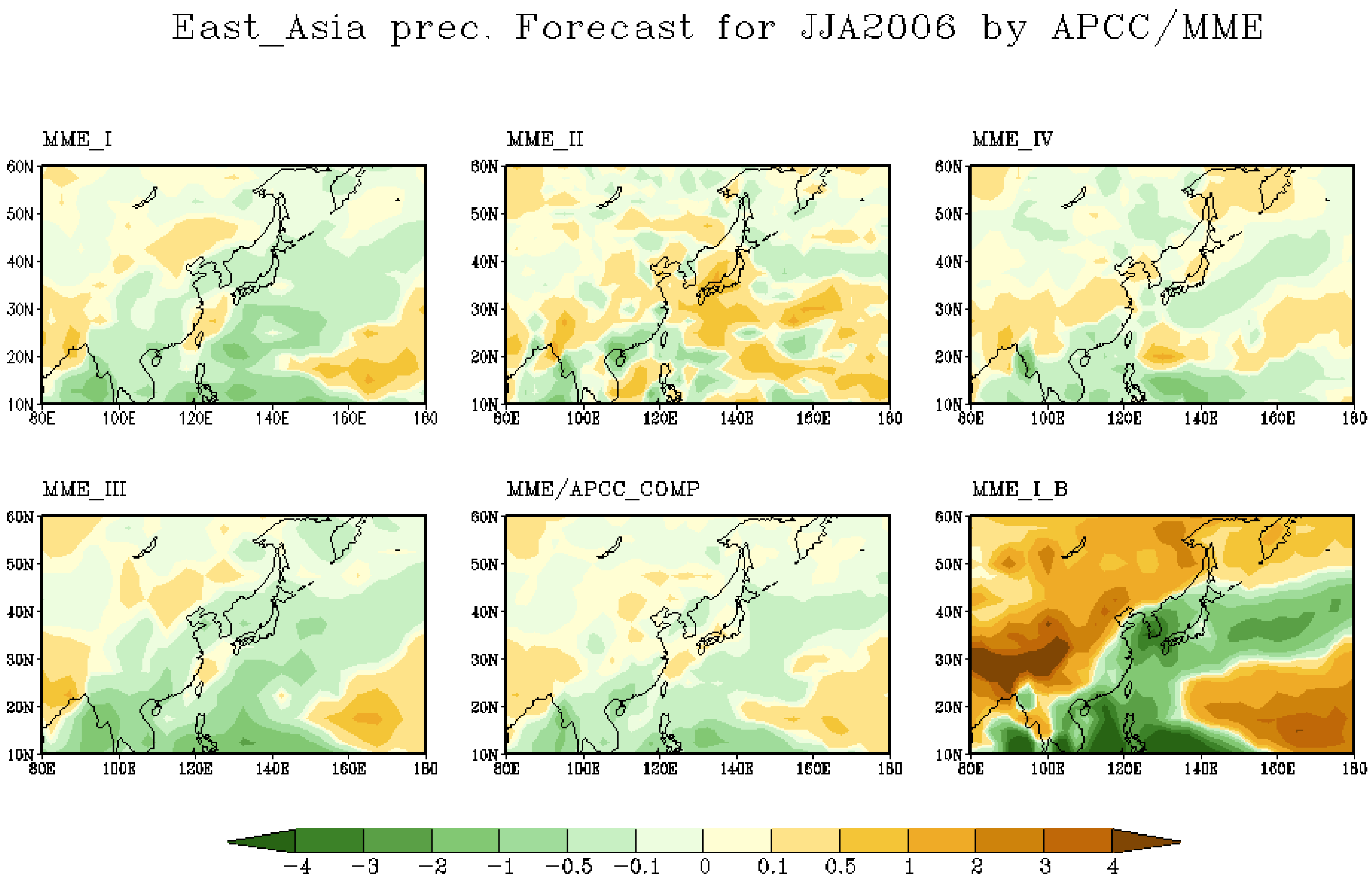
$$\widetilde{O}(t) = \sum_n \alpha_{i,n}\widetilde{F}_{i,n}(t) + \varepsilon_{i,n}(t)\,.$$

$$C_{n,m} = \widetilde{F}'_n(t)\widetilde{F}'_m(t)\,,$$

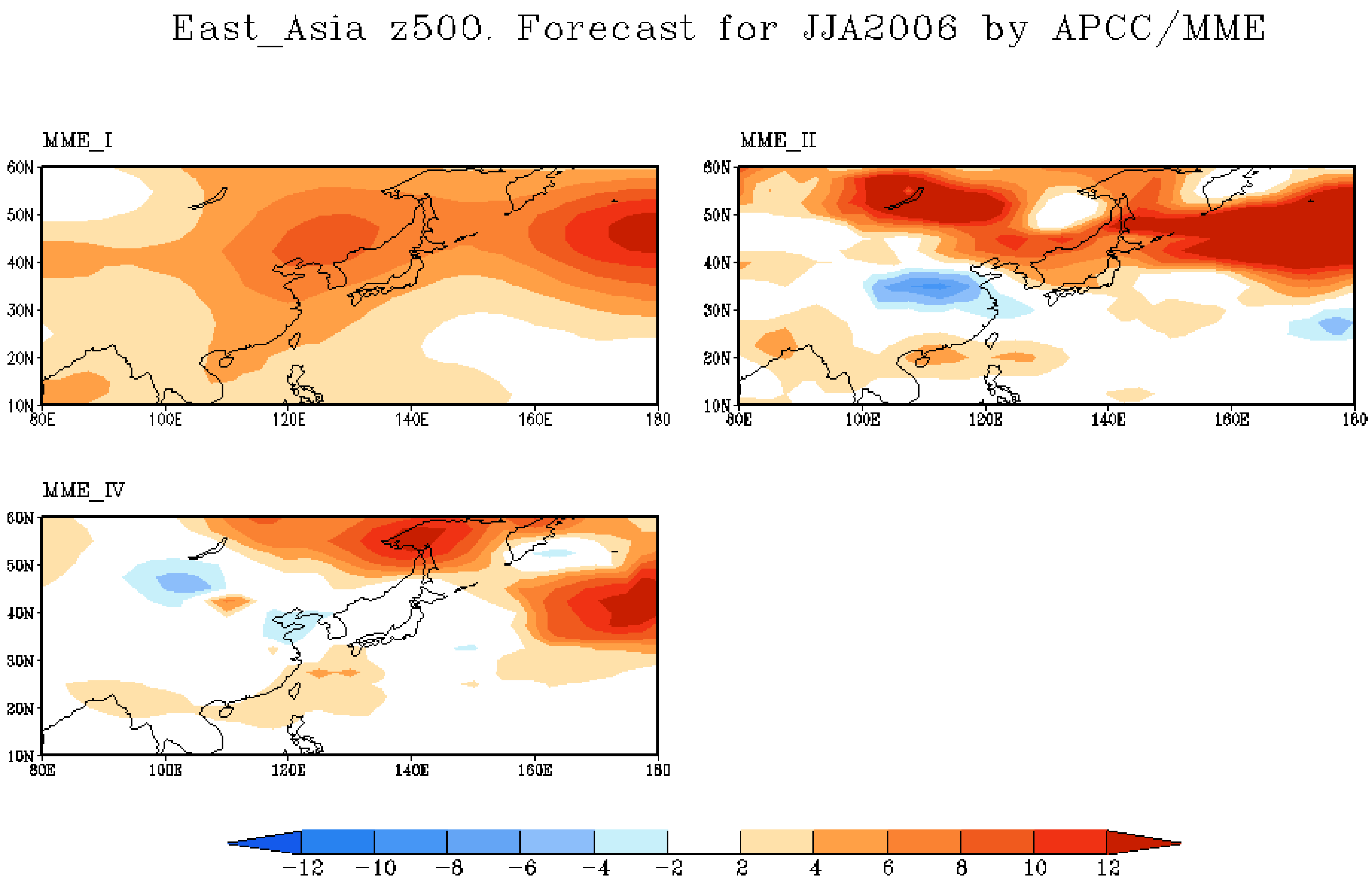
$$\widetilde{F}_i^{reg}(T) = \sum_n \alpha_{i,n}\widetilde{F}_{i,n}(T)\,.$$

$$F_i^{syn}(x,T) = \sum_n \widetilde{F}_{i,n}^{reg}(T) \cdot \phi_n(x)$$

East Asia Precipitation



East Asia 500hPa Geopotential Height



4. Verification

1. Introduction

2. Series

3. Fields

4. WMO SVS

5. Cross Validation

Notations:

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SKILL METRICS

Time series:

Pearson Correlation Coefficient

$$r = \frac{\sum_{i=1}^{N} x_i y_i}{\sqrt{\sum_{i=1}^{N} x_i^2 \sum_{i=1}^{N} y_i^2}}$$

Main restriction: iid

x and y are Normally distributed

Statistic

$$t^* = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}}$$

has Student's *t* distribution with *N*-2 degrees of freedom

x – model hindcast anomaly
y – observed anomaly

SKILL METRICS

Time series:

Spearman Correlation $$sr = 1 - \frac{6\sum_{i=1}^{N}(rank(x_i) - rank(y_i))^2}{N(N^2 - 1)}$$

Does not need Normal distribution of x and y
but is sensitive to serial correlation

Statistic $t^* = sr / \sigma_{sr}$ where $\sigma_{sr} = \frac{1}{\sqrt{N-1}}$

has Student's *t* distribution with *N*-2 degrees of freedom

x – model hindcast anomaly
y – observed anomaly

Time series:

Brier Skill Score

$$BSS = \left(1 - \frac{\sum_{i=1}^{N}(x_i - y_i)^2}{\sum_{i=1}^{N} y_i^2}\right)$$

x – model hindcast anomaly
y – observed anomaly

Time series:

Heidke Skill Score

$$HSS = (P_c - (P_e^2 + (1 - P_e)^2)) / (1 - (P_e^2 + (1 - P_e^2)))$$

x – model hindcast anomaly
y – observed anomaly

Pc – probability of correct anomaly in the hindcast
(number of points/years when x*y>=0 divided by N)
Pe – climatological probability of correct anomaly

Time Series:

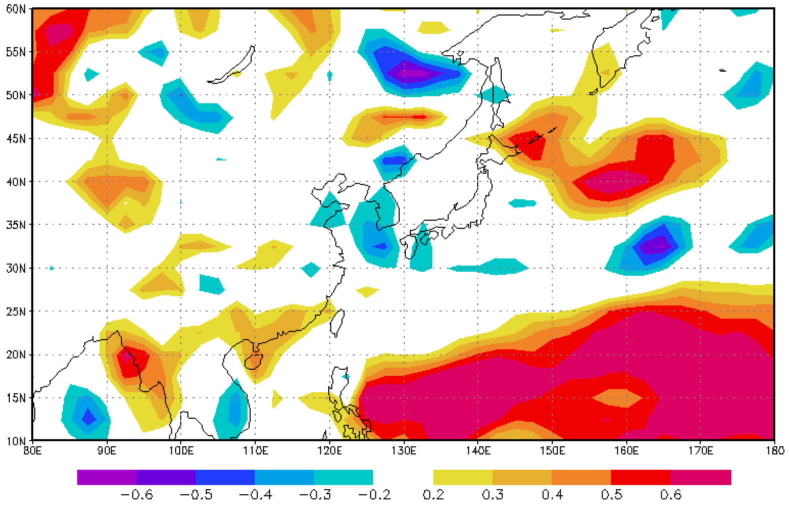
Root mean squared error

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(x_i - y_i)^2}{N}}$$

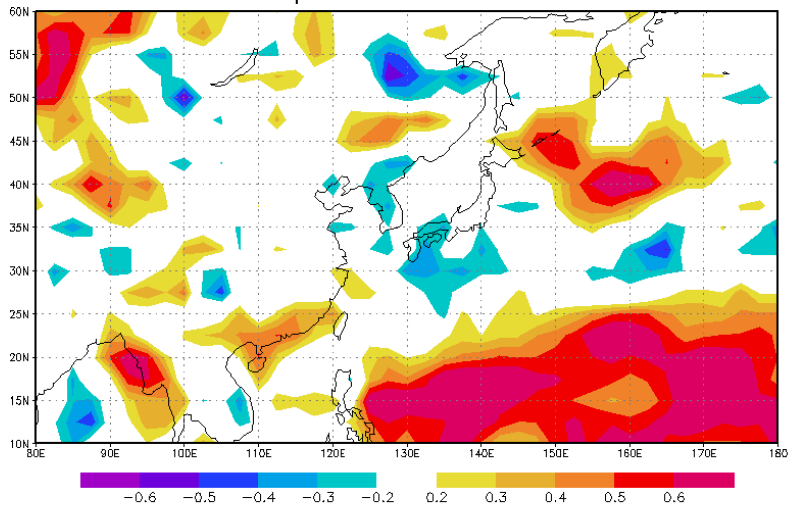
SKILL METRICS

SIMPLE COMPOSITE (MME 1)

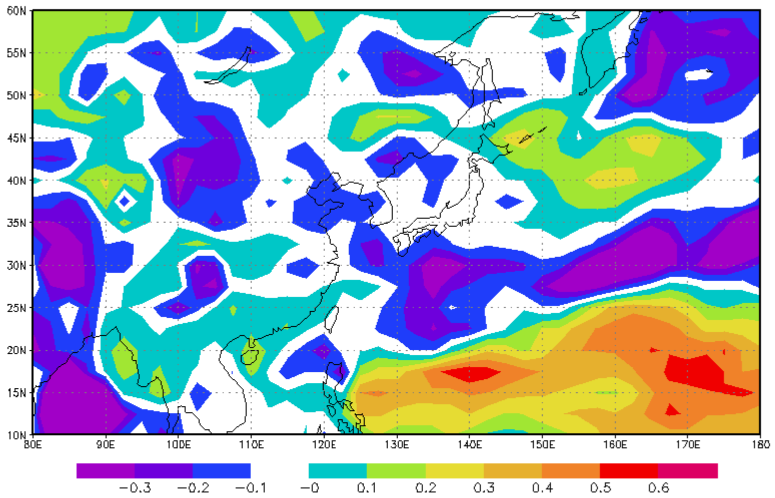
Correlation



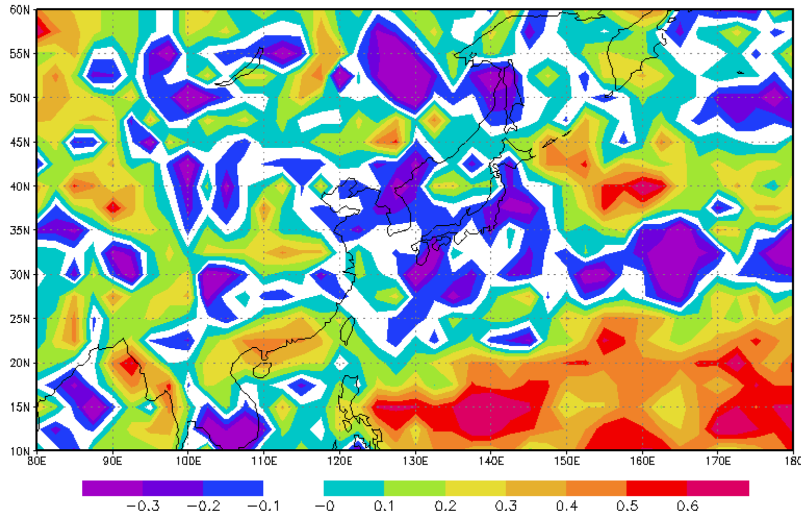
Spearman Correlation



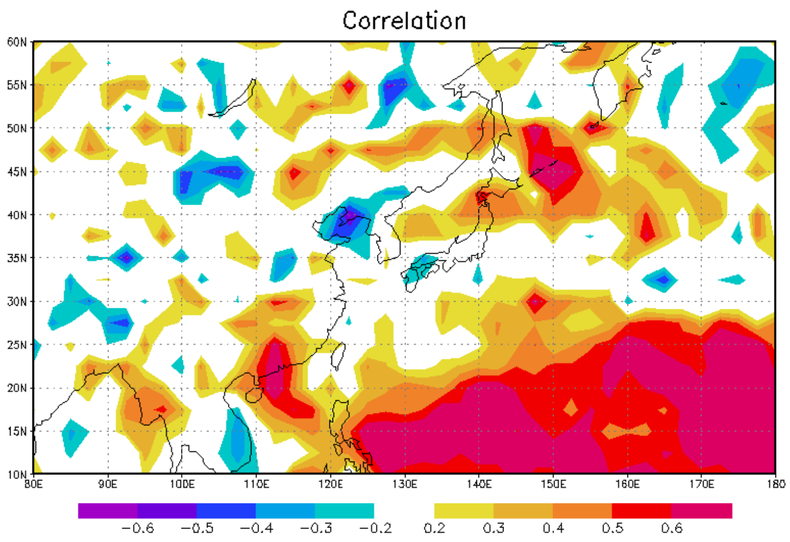
Brier Skill Score

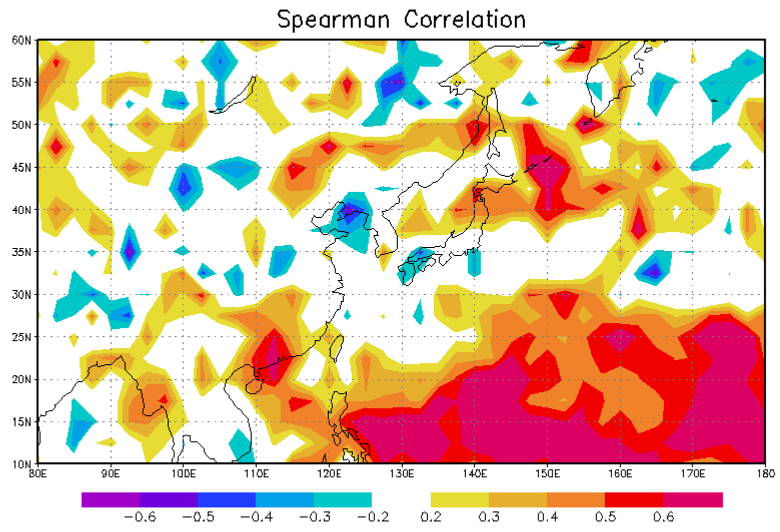


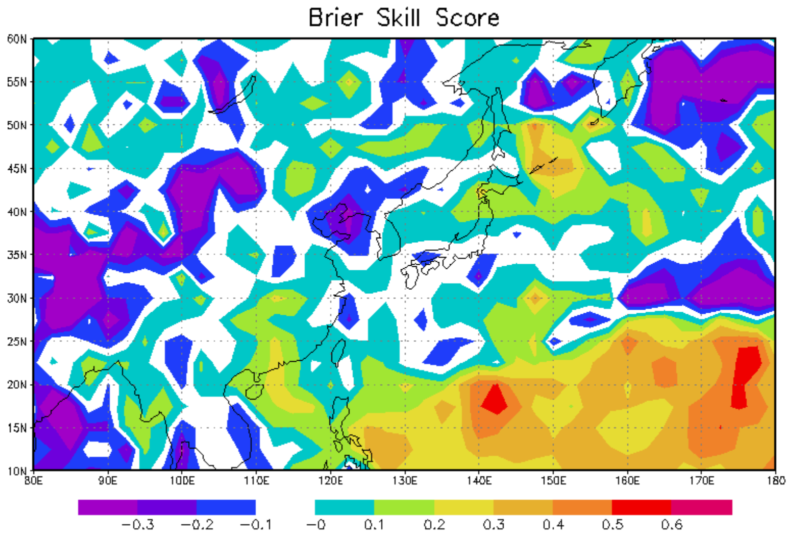
Heidke Scill Score

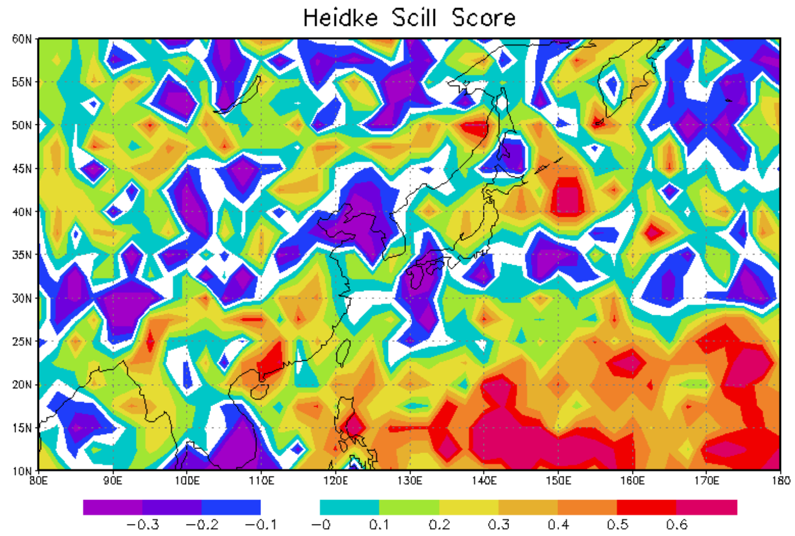


COMPOSITE of SPATIALLY CORRECTED MODELS (Method 4)









SKILL METRICS

Fields:

Brier Skill Score

$$BSS = (1 - \frac{\sum_{i=1}^{N}(x_i - y_i)^2}{\sum_{i=1}^{N} y_i^2})$$

x – model hindcast anomaly in a given year
y – observed anomaly in a given year

SKILL METRICS

Fields:

Heidke Skill Score

$$HSS = (P_c - (P_e^2 + (1 - P_e)^2))/(1 - (P_e^2 + (1 - P_e^2)))$$

x – model hindcast anomaly in a given year
y – observed anomaly in a given year

Pc – probability of correct anomaly in the hindcast
(number of points/years when x*y>=0 divided by N)
Pe – climatological probability of correct anomaly

Fields:

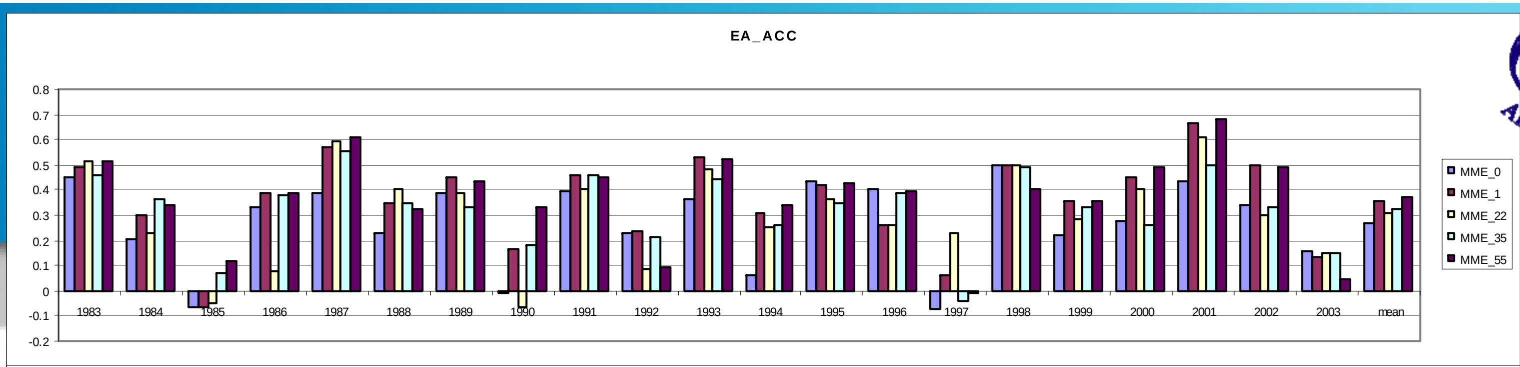
Anomaly correlation coefficient

$$ACC = \frac{\sum_{i=1}^{N} x_i y_i}{\sqrt{\sum_{i=1}^{N} x_i^2 \sum_{i=1}^{N} y_i^2}}$$

Fields:

Root mean squared error

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(x_i - y_i)^2}{N}}$$


EA_ACC
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
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mean
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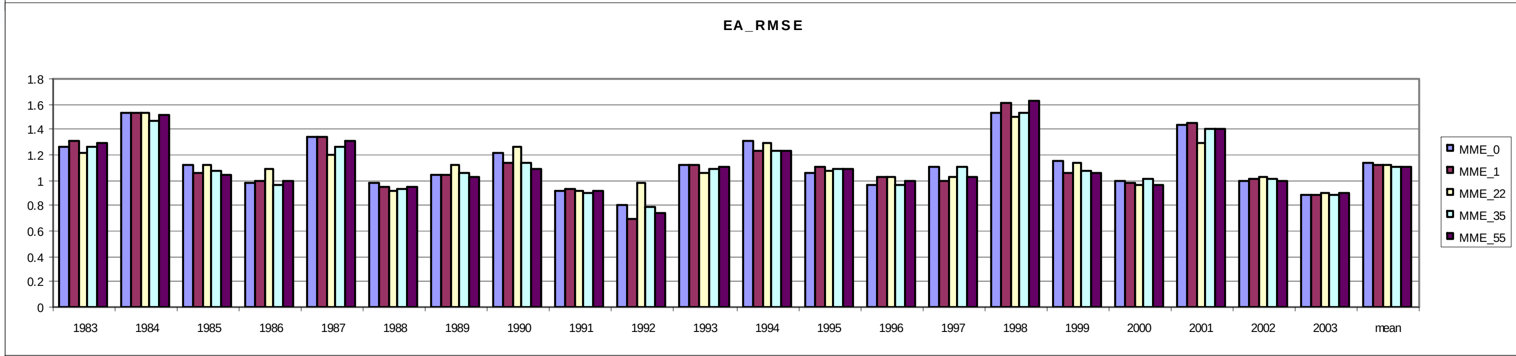
MME_35
MME_55


EA_RMSE
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1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
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2002
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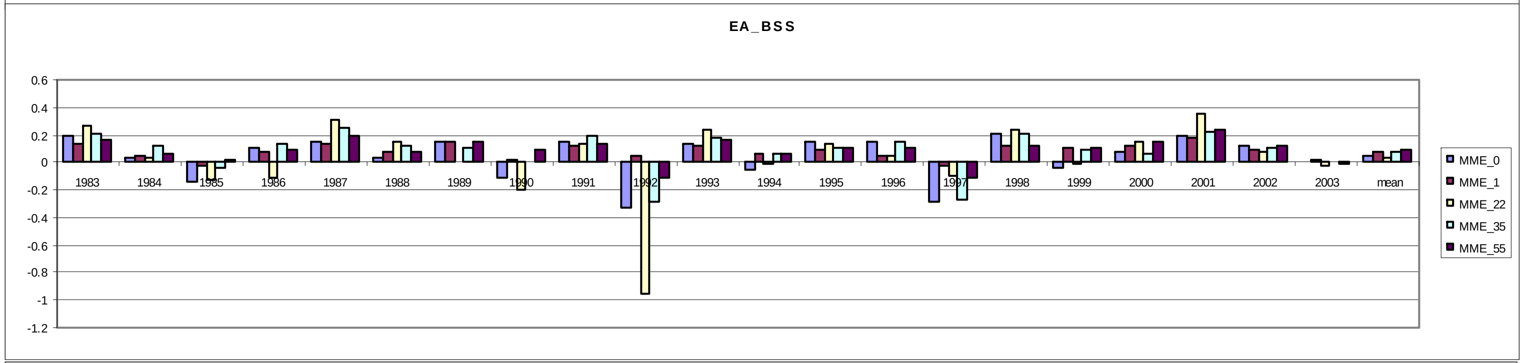
mean
MME_0
MME_1
MME_22
MME_35
MME_55


EA_BSS
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
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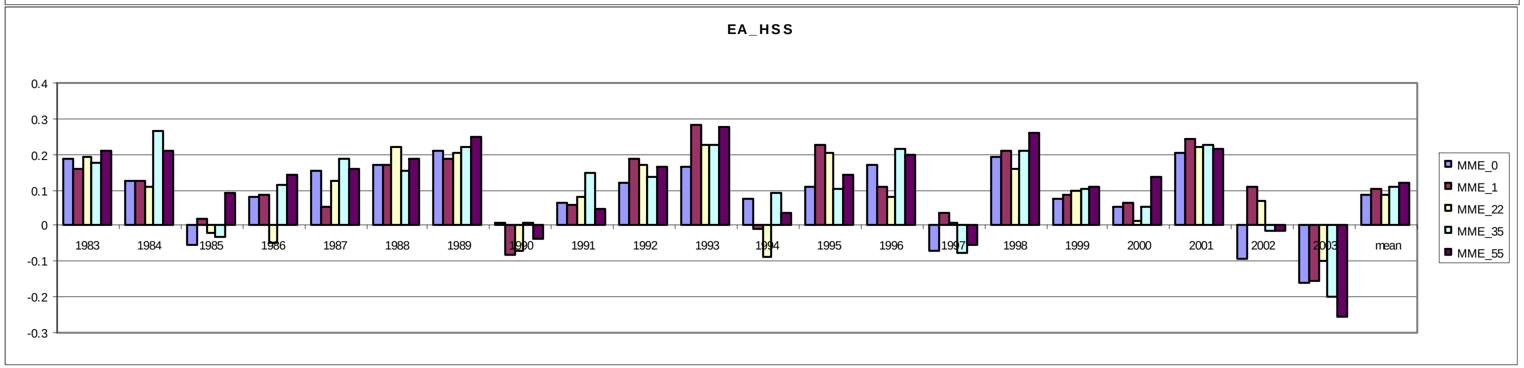
MME_35
MME_55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0.3
0.2
0.1
0
-0.1
-0.2
-0.3
1983
1984
1985
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1987
1988
1989
1990
1991
1992
1993
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mean
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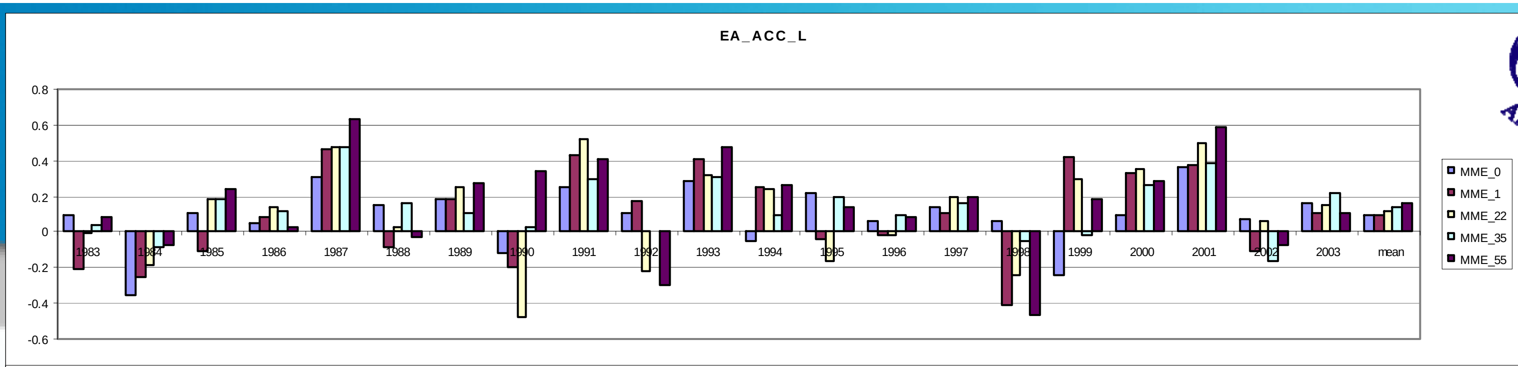
MME_22
MME_35
MME_55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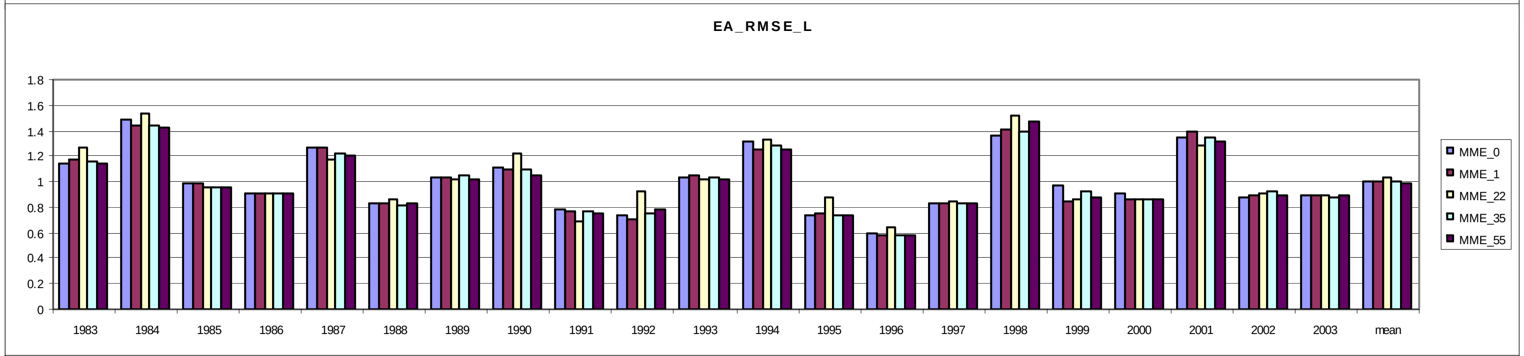
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 mean
MME_0
MME_1
MME_22
MME_35
MME_55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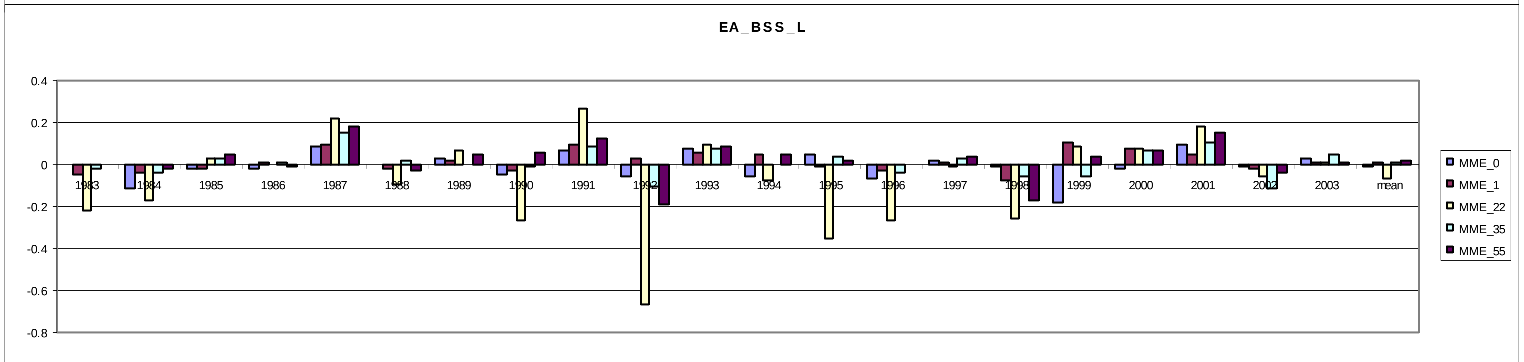
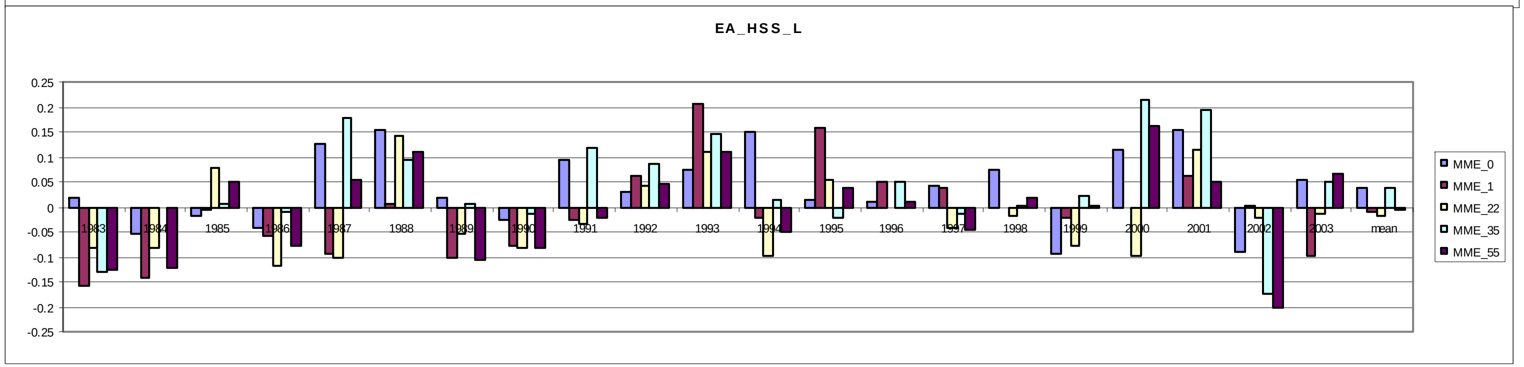
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 mean
MME_0
MME_1
MME_22
MME_35
MME_55


EA_BSS_L
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 mean
MME_0
MME_1
MME_22
MME_35
MME_55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0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 mean
MME_0
MME_1
MME_22
MME_35
MME_55

SKILL METRICS

WMO SVS:

$$MSSS_j = \left\{ 2\frac{s_{fj}}{s_{xj}} r_{fxj} - \left(\frac{s_{fj}}{s_{xj}}\right)^2 - \left(\frac{\left[\bar{f}_j - \bar{x}_j\right]}{s_{xj}}\right)^2 + \frac{2n-1}{(n-1)^2} \right\} \Bigg/ \left\{ 1 + \frac{2n-1}{(n-1)^2} \right\}$$

$$MSSS = 1 - \frac{\sum_j w_j MSE_j}{\sum_j w_j MSE_{cj}}$$

WMO SVS:

$$MSE_{cj} = \frac{n-1}{n} s_{xj}^2$$

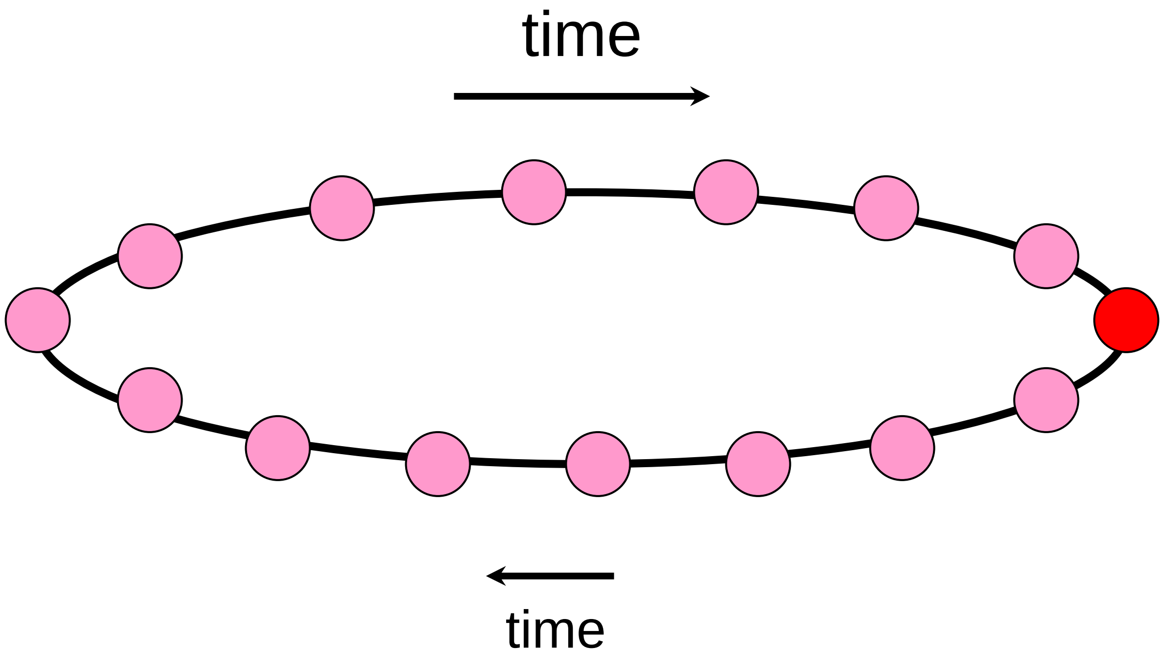
$$MSE_j = \frac{1}{n} \sum_{i=1}^{n} \left(f_{ij} - x_{ij} \right)^2$$

$$\bar{x}_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij}, \; \bar{f}_j = \frac{1}{n} \sum_{i=1}^{n} f_{ij}$$

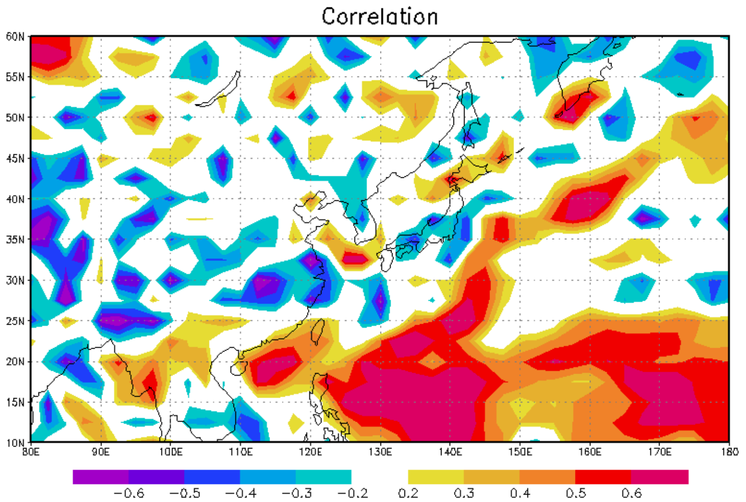
$$s_{xj}^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(x_{ij} - \bar{x}_j \right)^2, \; s_{fj}^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(f_{ij} - \bar{f}_j \right)^2$$

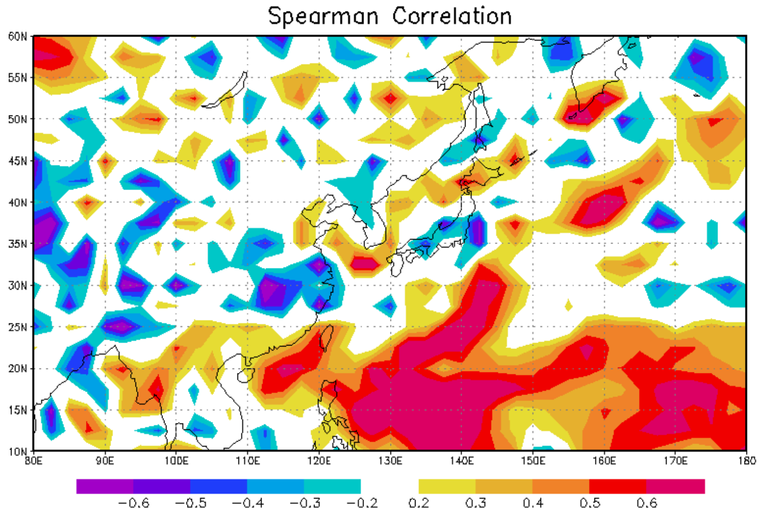
SKILL METRICS

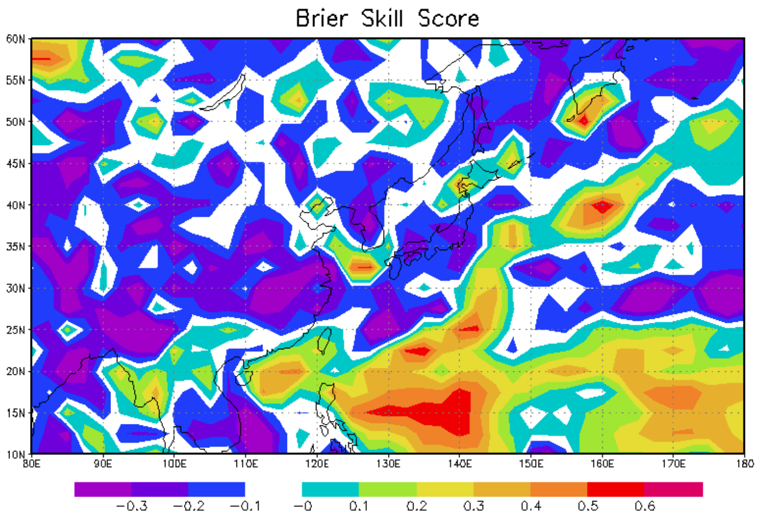
Cross-Validation

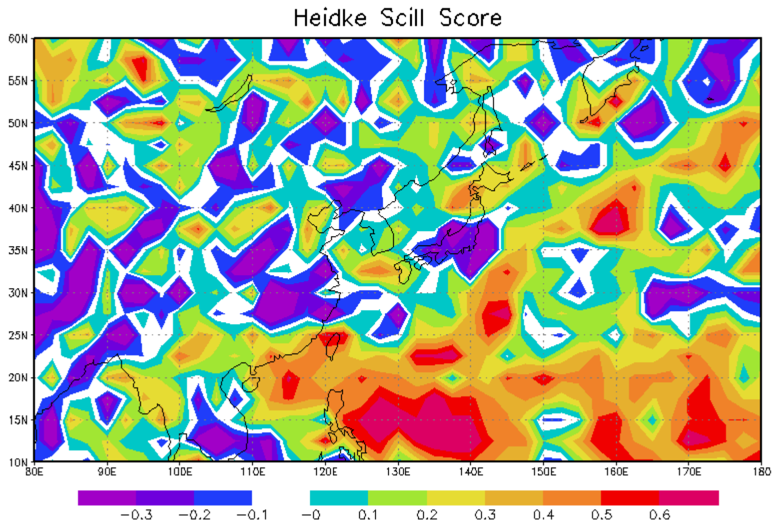


SUPERENSEMBLE 2 SVD MODES

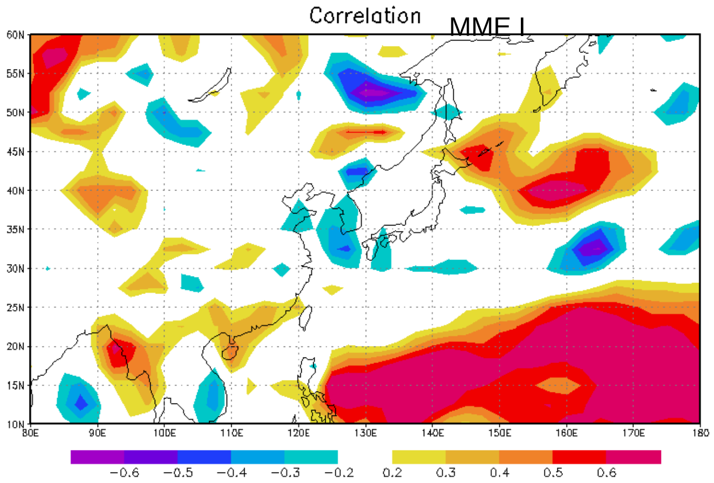


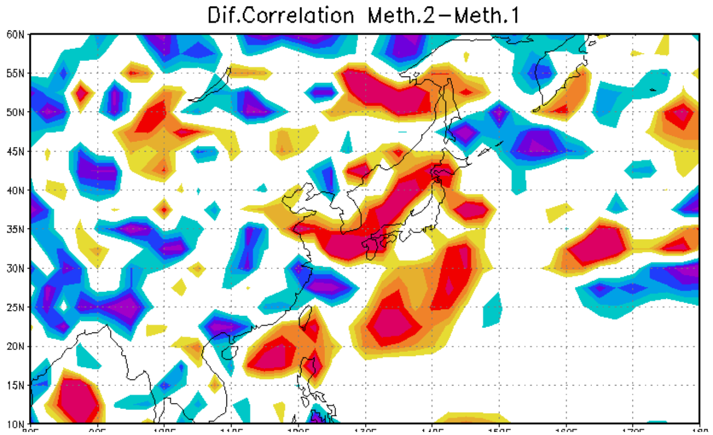


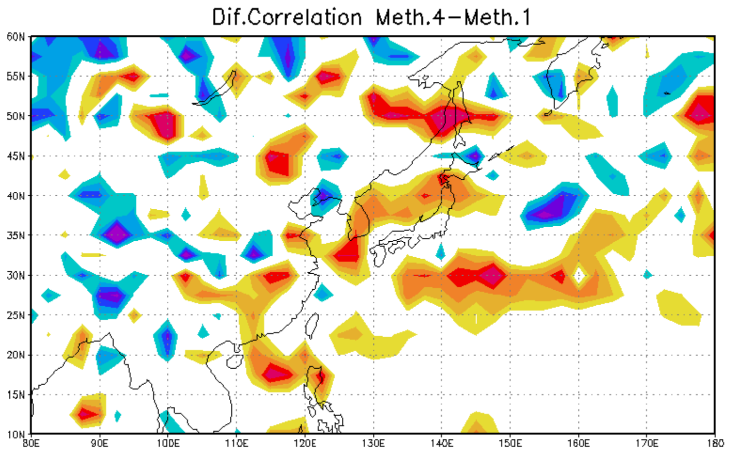


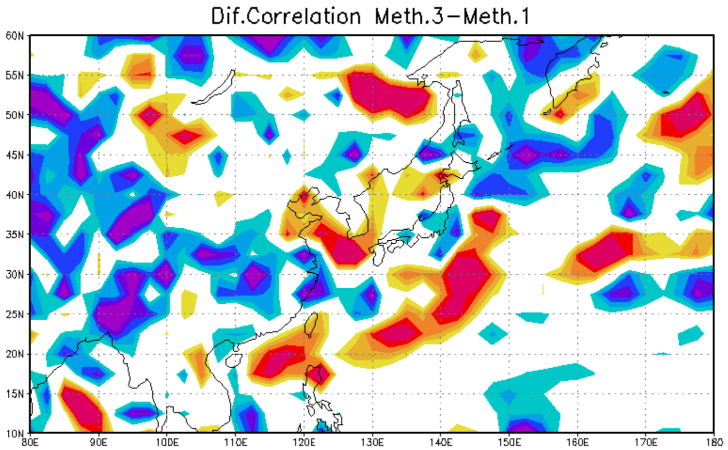


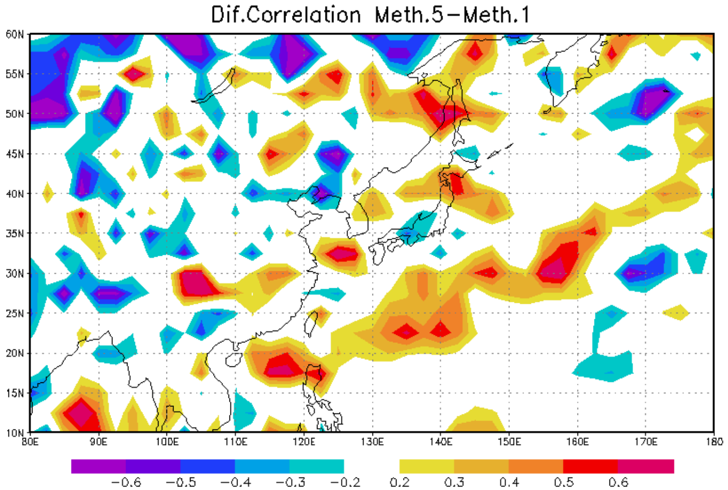
DIFFERENCE in CORRELATIONS
Improvement and deterioration as against MME I
(qualitative assessment)



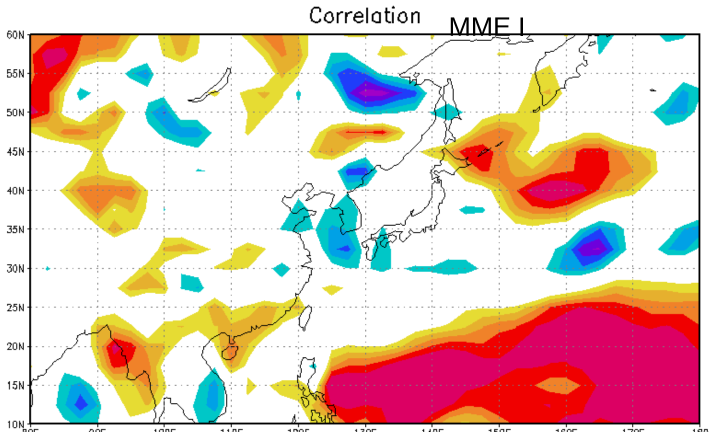


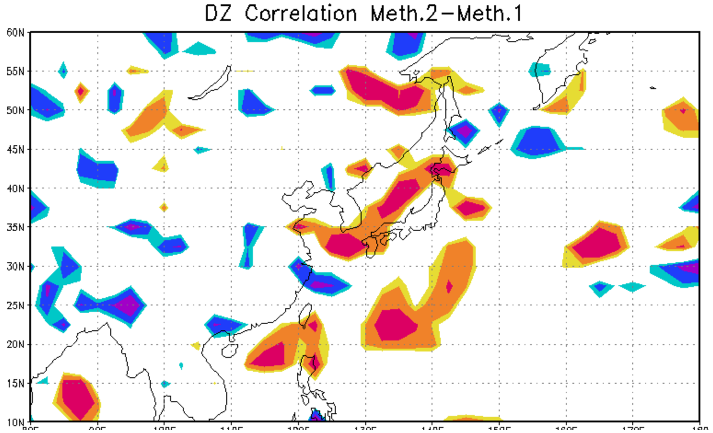


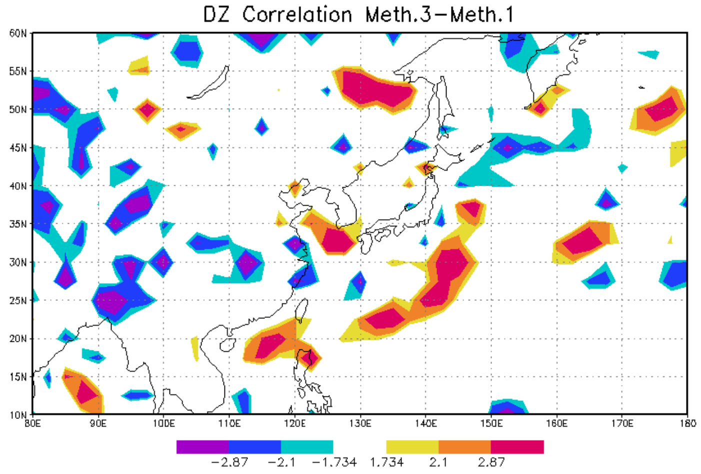


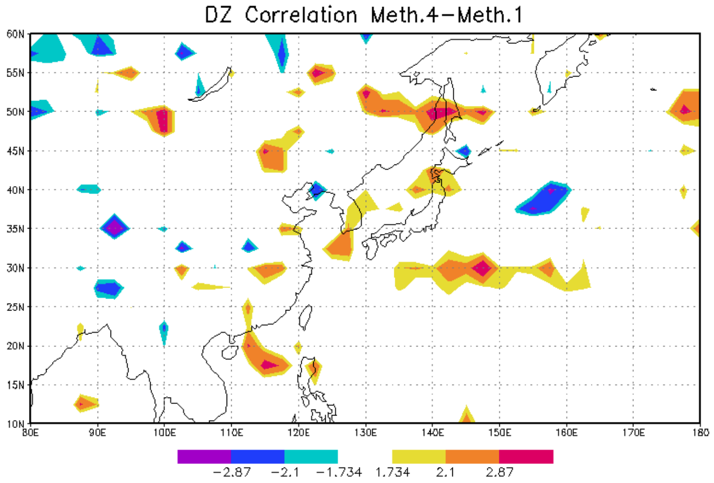


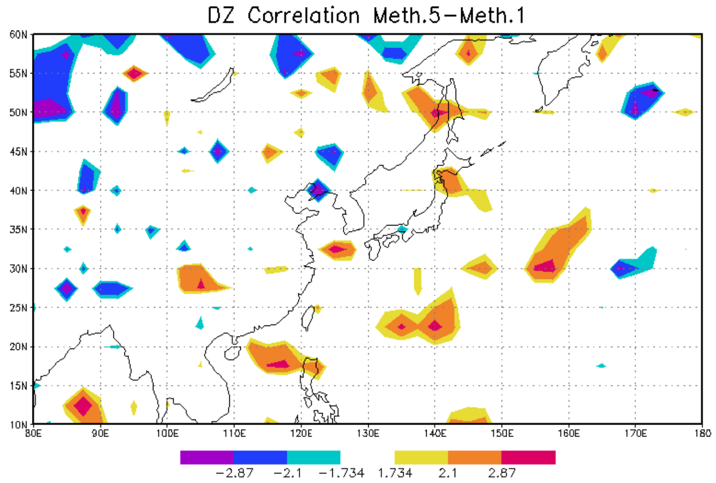
Areas of
Statistically significant (10%, 5%, 1%)
DIFFERENCE in CORRELATIONS
as against MME I
$|t^*=(z_i-z_1)/\sigma_z| >$ criterion



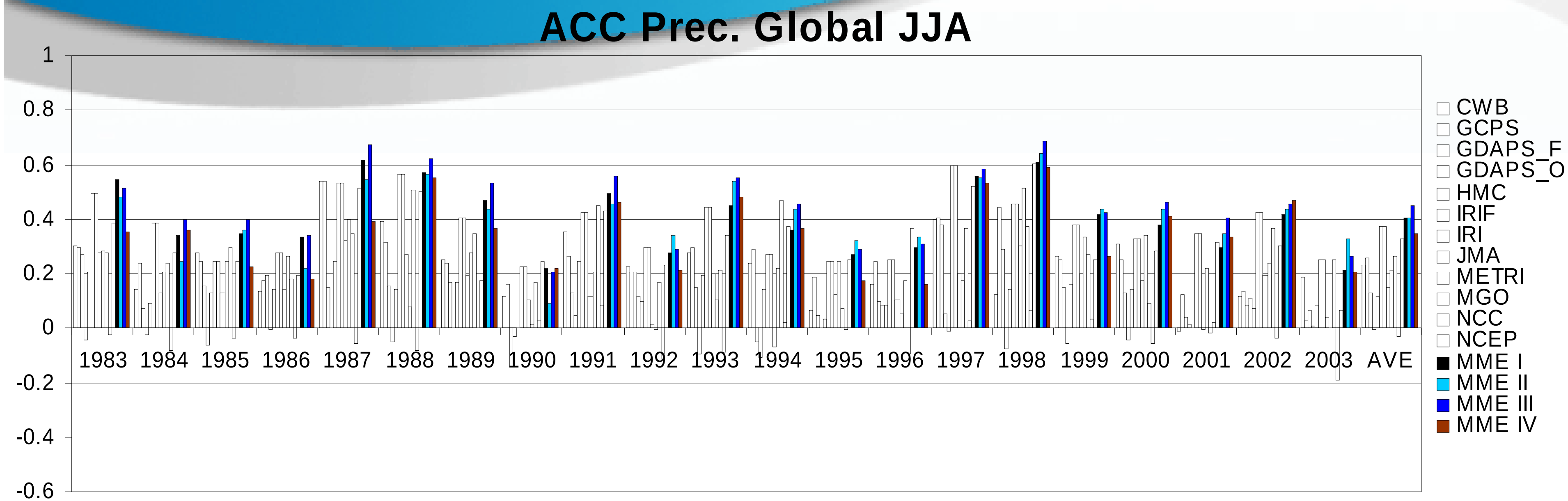


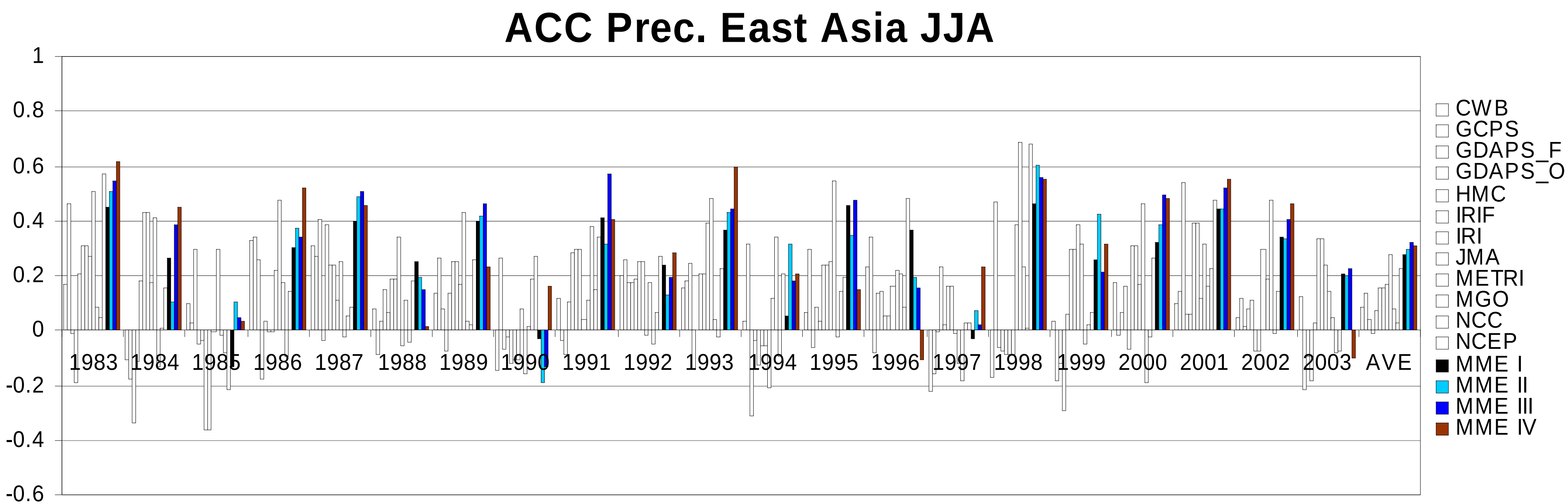






Hindcast Skill





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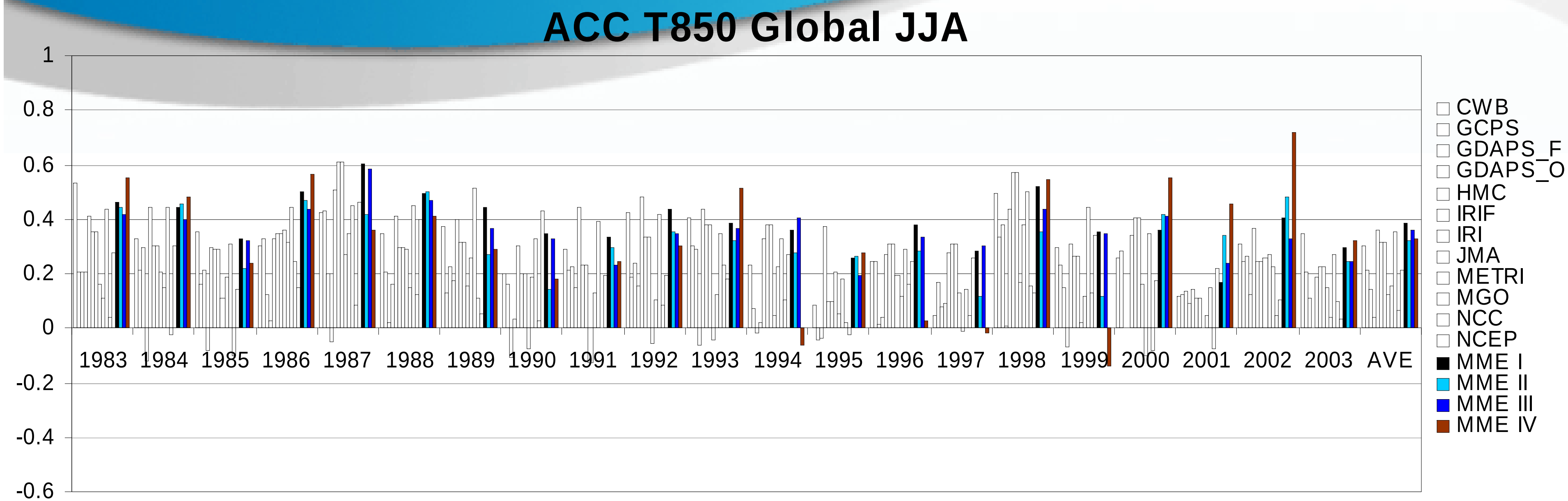

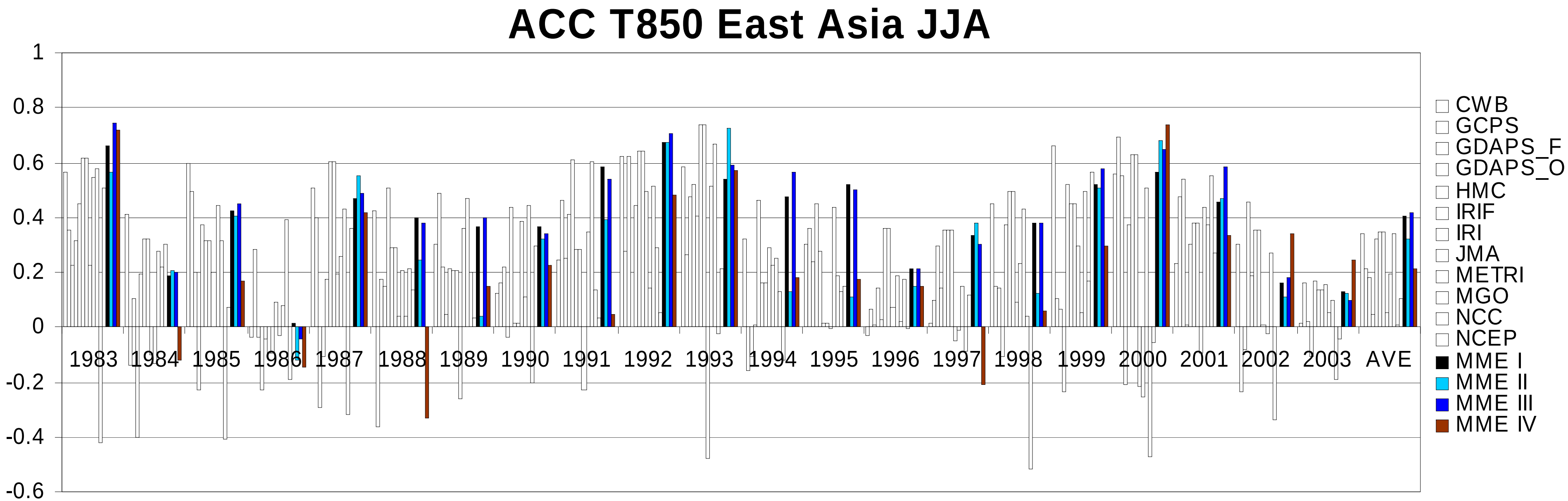

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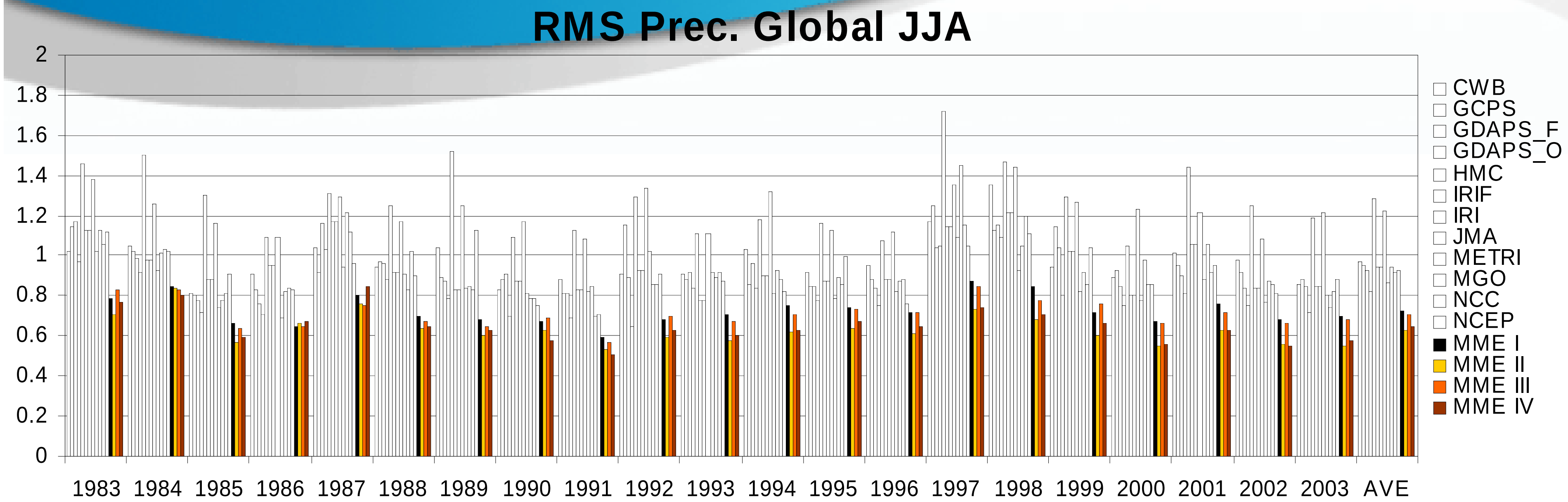

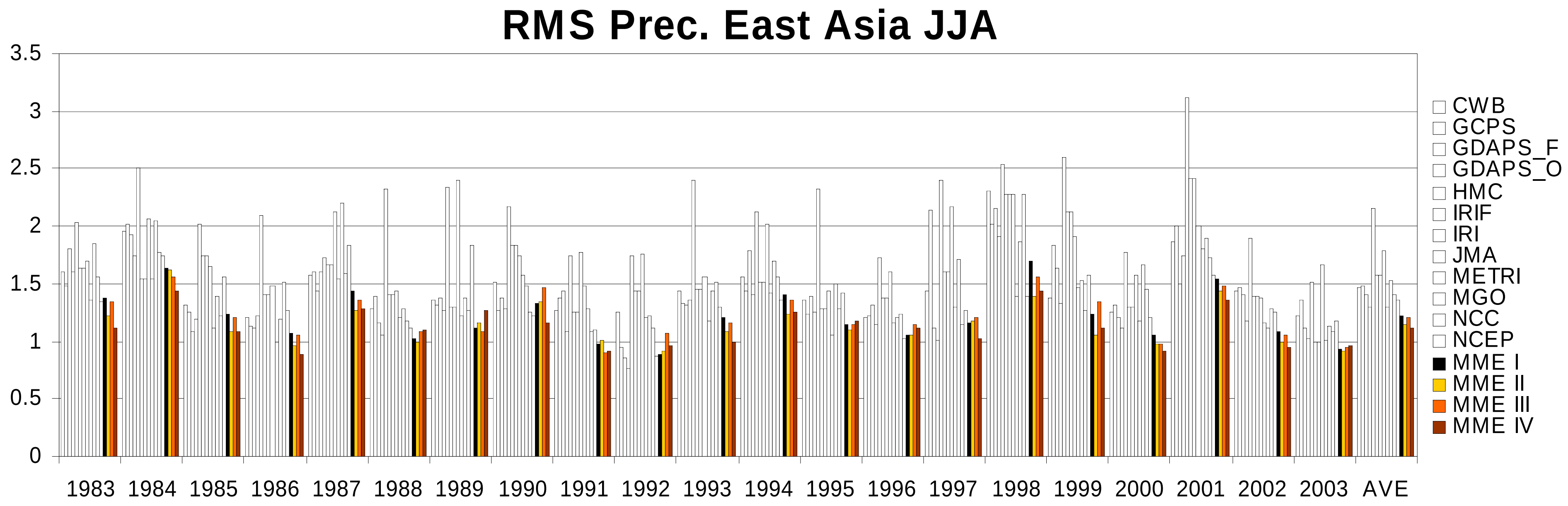

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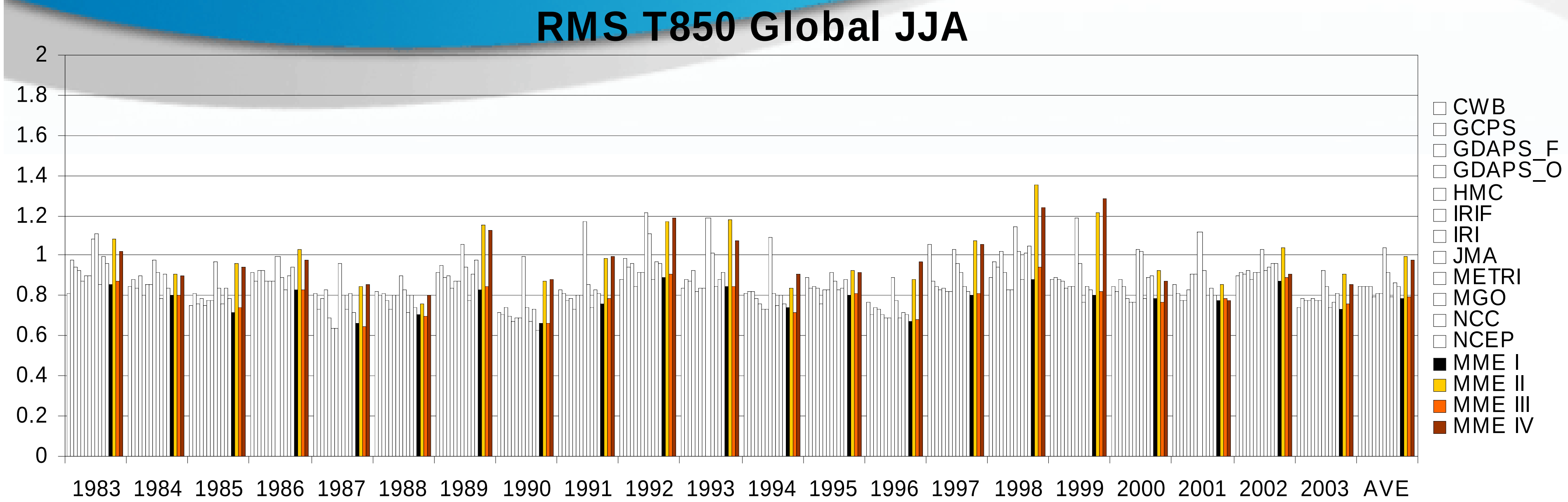

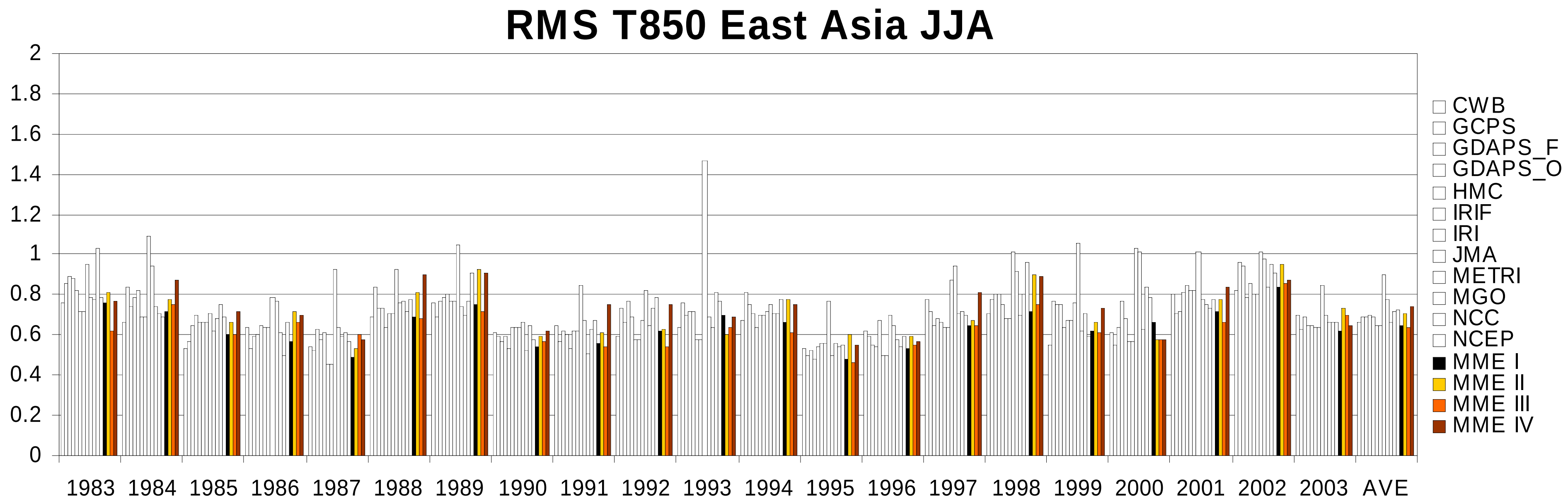

Conclusion

1. Multimodel ensemble outperforms single model as a rule

2. The "best model" may outperform ensemble in a particular point, however each point has its own best model

3. Choice of the best method to blend the models depends upon particular purpose